APW

Core Instructional Program

Grades 6 - 12

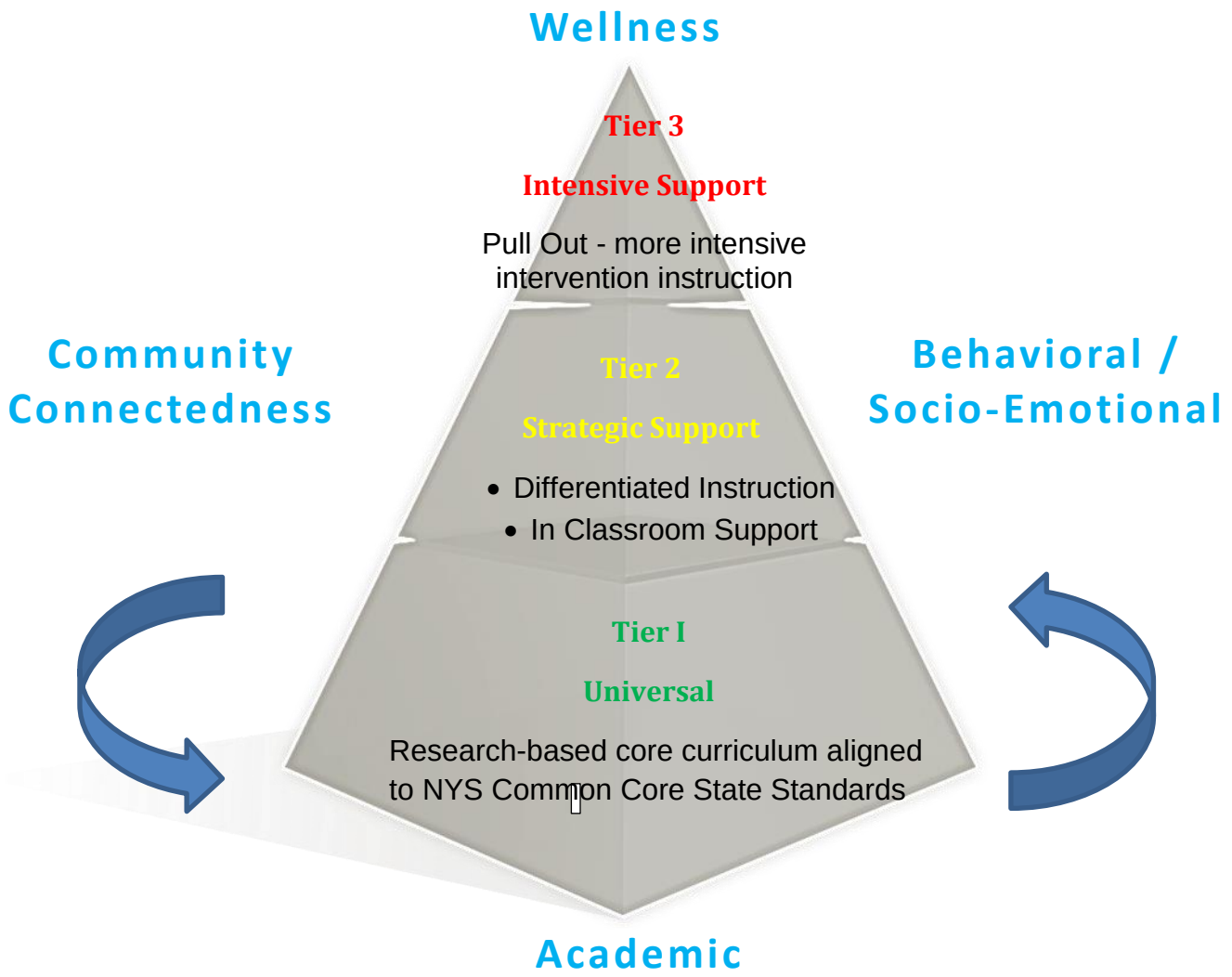
Response to Intervention

Response to Intervention (RTI) Defined

Response to Intervention integrates assessment and intervention within a multi-level prevention system to maximize student achievement. With RTI, schools can use data to identify students at risk for poor learning outcomes, monitor student progress, provide evidence-based interventions and adjust the intensity and nature of those interventions depending on a student's responsiveness, and identify students with learning disabilities (NCRTI, 2010).

RTI - A Multi-Tiered Prevention Framework

RTI serves as a multi—tiered prevention framework/model with increasing levels or tiers of instructional support. Within the Altmar Parish Williamstown Central School District a three-tiered model is will be used. The graphic presented below provides a visual illustration of the district's RTI model. Further information for each tier follows the graphic.



Tier One: Appropriate Instruction

Tier 1 is identified as the core instruction program provided to all students. Research-based instruction and positive universal behavioral supports are part of the core program. The APW core program (Tier 1) should minimally include:

- Research-based core curriculum aligned to NYS Common Core State Standards
- Appropriate instruction and research-based instructional interventions that meets the needs of at least 80 percent of all learners. Appropriate instruction in reading means scientific research-based reading programs that include explicit and systematic instruction in reading foundational skills (print concepts, phonological awareness, phonics, word recognition) vocabulary development, reading fluency and reading comprehension strategies. Appropriate instruction in mathematics means research-based instruction programs that include instruction in problem solving, arithmetic skills and fluency, conceptual knowledge/number sense, and reasoning ability to apply mathematics to real life situations and contexts.
- Differentiated, flexible groups determined by universal screening and progress monitoring data (refer to Section 3) for application of skills, re-teaching, additional practice and/or challenge activities

- **Core Curricular Daily Instructional Time Allotments (K-6)**
    - It is expected that daily instructional time be devoted to each of the four core content areas, English Language Arts, Mathematics, Science and Social Studies. These daily time allotments should continue for the entire school year.
    - It is suggested that at the elementary levels, ELA be delivered in a best practice model of approximately 110 minute per day. An additional 30 minutes of a literacy block is used to implement needed literacy interventions to support students specific literacy needs
    - Progress monitoring in English and Mathematics once per month
    - Universal screening administered three times per year
    - Use of benchmark data and curriculum based measures to inform instruction
    - Summative assessment to determine student mastery and grade
    - Consistent communication with parents regarding student progress and academic needs
    - Parents of all students should be notified of school-wide screening results

- **Core Curricular Daily Instructional Time Allotments (7-8)**
    - It is expected that a minimum of 200 minutes of weekly instructional time be devoted to each of the four core content areas, English Language Arts, Mathematics, Science and Social Studies. These weekly time allotments should continue for the entire school year. In-class supports as determined by progress monitoring
    - Progress monitoring in English and Mathematics once per month
    - Universal screening administered three times per year

- Use of benchmark data and curriculum based measures to inform instruction
- Summative assessment to determine student mastery and grade
- Consistent communication with parents regarding student progress and academic needs
- Parents of all students should be notified of school-wide screening results

Considerations for Appropriate Instruction:

- **Culturally Responsive Instruction -** Being a Culturally Responsive Educator means that one is aware, sensitive and actively integrating the culture of one's students into daily lessons and interactions. A culturally responsive teacher embodies various characteristics, the most important being caring, engaging and seeks high expectations from his/her students. In regards to the RTI pyramid of intervention, Culturally Responsive teaching is for 100% of the student population, or is integrated in Tier I. Teachers who embrace its philosophies and practices may experience fewer misunderstandings of student behaviors that may inappropriately move preventative actions into Tiers II and III of RTI. Providing linguistic and cultural relevance to student learning and building authentic relationships is the core of Culturally Responsive Teaching
- **Students with Disabilities --** Appropriate core instruction for students with disabilities must be consistent with the student's Individualized Educational Plan (IEP).

Tier Two: Supplemental Intervention

Tier 2 supplemental instructional intervention is provided **in addition to, and not in place of,** the standards-based curriculum received in Tier 1. The intervention focus is on the areas of student need or weakness that are identified in the screening, assessment or progress monitoring of student progress in core instruction (Tier 1). The determination of which interventions will be provided to an individual student is made either by a problem-solving process or a standard treatment protocol. Tier 2 instruction is typically delivered in a small group (recommended maximum of 9 students) of students often arranged according to similar instructional needs. The recommended length of time a student spends in the second tier of intervention will vary up to approximately twenty weeks depending on such factors as the skill and the level of the student's responsiveness to the intervention. Approximately 10 percent of students in a particular grade level should be provided Tier 2 intervention. In cases when the number of students not meeting standards exceeds 10%, it is recommended that schools utilize contextual norms to prioritize students who require additional instruction beyond differentiation in the core instruction.

Tier 2 should minimally include:

- Alignment to Tier 1 core curriculum with supplemental supports
- Research-based instructional materials selected to match student need based on progress monitoring and other data
- Direct, Scaffold instruction
- Frequent opportunities for students to apply their thinking
- Small, homogenous groups
- Three days per week minimum
- 20-30 minutes per session
- 4-8 week intervals for up to 20 weeks
- Group size up to 9 students with size of group dependent on program/strategy recommendations
- Progress monitoring at a minimum of every two weeks
- Periodic fidelity checks are required to ensure that the delivery of instruction was provided in the way it was intended.

Tier Three: Intensive Intervention

Tier 3 intervention is designed for those students who demonstrate insufficient progress in Tier 2. Tier 3 is typically for approximately one to five percent of students in a grade level who require more intensive intervention instruction in addition to their core instruction. This tier provides greater individualized instruction in a smaller group setting (with a maximum of 5 students) with more time, duration and more frequent progress monitoring.

Tier 3 should minimally include:

- An intensive intervention program aligned with Tier 1 core curriculum
- Research-based instructional materials to meet individualized needs
- For secondary students, a specific course may be included during which intensive intervention is provided
- Individual or small homogeneous groups
- Group size is a maximum of 5 students; size dependent on program or strategy recommendations.
- Five day per week
- 30-60 minutes per sessions
- 6-8 week intervals up to 20 weeks
- Weekly progress monitor
- Periodic fidelity checks are required to ensure that the delivery of instruction was provided in the way it was intended.

If the student is determined to be making substandard progress in such areas of study, instruction shall be provided that is tailored to meet the student's individual needs with increasingly intensive levels of targeted intervention and instruction.

School districts must provide written notification to parents when a student requires an intervention beyond that which is provided to the general education classroom. Such notification shall include: information about the performance data that will be collected and the general education services that will be provided; strategies for increasing the student's rate of learning; and the parents' right to request an evaluation by the Committee on Special Education to determine whether the student has a disability.

A RTI process as described above will meet the section 117.3 requirements to ensure a student's progress toward meeting the State's standards.

English Language Arts

Core Instructional Program

New York State Common Core Learning Standards for

English Language Arts and Literacy

College and Career Readiness Anchor Standards for Reading

The Common Core Reading Standards for Literacy require the explicit instruction in reading strategies across content areas. The goal of these standards is to develop independent, strategic readers. The College and Career Ready framework is intended to support an interdisciplinary approach across curriculums.

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, and ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

Responding to Literature

11. Respond to literature by employing knowledge of literary language, textual features, and forms to read and comprehend, reflect upon, and interpret literary texts from a variety of genres and a wide spectrum of American and world cultures.

Reading Standards for Literature 6–12 UNPACKED

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details		
1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.	2. Determine a theme or central idea of a text and analyze its development over the course of the text; provide an objective summary of the text.	2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text.
3. Describe how a particular story's or drama's plot unfolds in a series of episodes as well as how the characters respond or change as the plot moves toward a resolution.	3. Analyze how particular elements of a story or drama interact (e.g., how setting shapes the characters or plot).	3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of rhymes and other repetitions of sounds (e.g., alliteration) on a specific verse or stanza of a poem or section of a story or drama	4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot.	5. Analyze how a drama's or poem's form or structure (e.g., soliloquy, sonnet) contributes to its meaning.	5. Compare and contrast the structure of two or more texts and analyze how the differing structure of each text contributes to its meaning and style.

Reading Standards for Literature 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Craft and Structure		
6. Explain how an author develops the point of view of the narrator or speaker in a text. a. Explain how an author's geographic location or culture affects his or her perspective.	6. Analyze how an author develops and contrasts the points of view of different characters or narrators in a text. a. Analyze stories, drama, or poems by authors who represent diverse world cultures.	6. Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor. a. Analyze full-length novels, short stories, poems, and other genres by authors who represent diverse world cultures.
Integration of Knowledge and Ideas		
7. Compare and contrast the experience of reading a story, drama, or poem to listening to or viewing an audio, video, or live version of the text, including contrasting what they "see" and "hear" when reading the text to what they perceive when they listen or watch.	7. Compare and contrast a written story, drama, or poem to its audio, filmed, staged, or multimedia version, analyzing the effects of techniques unique to each medium (e.g., lighting, sound, color, or camera focus and angles in a film).	7. Analyze the extent to which a filmed or live production of a story or drama stays faithful to or departs from the text or script, evaluating the choices made by the director or actors.
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics.	9. Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history.	9. Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently.

Reading Standards for Literature 6–12 – cont. UNPACKED

Responding to Literature		
11. Recognize, interpret, and make connections in narratives, poetry, and drama, ethically and artistically to other texts, ideas, cultural perspectives, eras, personal events, and situations. a. Self-select text based on personal preferences. b. Use established criteria to classify, select, and evaluate texts to make informed judgments about the quality of the pieces.	11. Recognize, interpret, and make connections in narratives, poetry, and drama, ethically and artistically to other texts, ideas, cultural perspectives, eras, personal events, and situations. a. Self-select text based on personal preferences. b. Use established criteria to classify, select, and evaluate texts to make informed judgments about the quality of the pieces.	11. Interpret, analyze, and evaluate narratives, poetry, and drama, artistically and ethically by making connections to: other texts, ideas, cultural perspectives, eras, personal events, and situations. a. Self-select text to develop personal preferences. b. Establish and use criteria to classify, select, and evaluate texts to make informed judgments about the quality of the pieces.

Reading Standards for Literature 6–12 – cont. UNPACKED

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grade 9-10 students:	Grade 11-12 students:
Key Ideas and Details	
1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
2. Determine a theme or central idea of a text and analyze in detail its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.	2. Determine two or more themes or central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to produce a complex account; provide an objective summary of the text.
3. Analyze how complex characters (e.g., those with multiple or conflicting motivations) develop over the course of a text, interact with other characters, and advance the plot or develop the theme.	3. Analyze the impact of the author's choices regarding how to develop and relate elements of a story or drama (e.g., where a story is set, how the action is ordered, how the characters are introduced and developed).
Craft and Structure	
4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language evokes a sense of time and place; how it sets a formal or informal tone).	4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (Include Shakespeare as well as other authors.)
5. Analyze how an author's choices concerning how to structure a text, order events within it (e.g., parallel plots), and manipulate time (e.g., pacing, flashbacks) create such effects as mystery, tension, or surprise.	5. Analyze how an author's choices concerning how to structure specific parts of a text (e.g., the choice of where to begin or end a story, the choice to provide a comedic or tragic resolution) contribute to its overall structure and meaning as well as its aesthetic impact.
6. Analyze a particular point of view or cultural experience reflected in a work of literature from outside the United States, drawing on a wide reading of world literature.	6. Analyze a case in which grasping point of view requires distinguishing what is directly stated in a text from what is really meant (e.g., satire, sarcasm, irony, or understatement).

Reading Standards for Literature 6–12 – cont. UNPACKED

Grade 9-10 students:	Grade 11-12 students:
Integration of Knowledge and Ideas	
7. Analyze the representation of a subject or a key scene in two different artistic mediums, including what is emphasized or absent in each treatment (e.g., Auden's "Musée des Beaux Arts" and Breughel's Landscape with the Fall of Icarus). a. Analyze works by authors or artists who represent diverse world cultures.	7. Analyze multiple interpretations of a story, drama, or poem (e.g., recorded or live production of a play or recorded novel or poetry), evaluating how each version interprets the source text. (Include at least one play by Shakespeare and one play by an American dramatist.) a. Analyze multiple interpretations of full-length works by authors who represent diverse world cultures.
8. (Not applicable to literature)	8. (Not applicable to literature)
9. Analyze how an author draws on and transforms source material in a specific work (e.g., how Shakespeare treats a theme or topic from Ovid or the Bible or how a later author draws on a play by Shakespeare).	9. Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics.
Range of Reading and Level of Text Complexity	
10. By the end of grade 9, read and comprehend literature, including stories, dramas, and poems, in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of By the end of grade 10, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 11, read and comprehend literature, including stories, dramas, and poems, in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 11–CCR text complexity band independently and proficiently
Responding to Literature	
11. Interpret, analyze, and evaluate narratives, poetry, and drama, aesthetically and ethically by making connections to: other texts, ideas, cultural perspectives, eras, personal events and situations. a. Self-select text to respond and develop innovative perspectives. b. Establish and use criteria to classify, select, and evaluate texts to make informed judgments about the quality of the pieces.	11. Interpret, analyze, and evaluate narratives, poetry, and drama, aesthetically and philosophically by making connections to: other texts, ideas, cultural perspectives, eras, personal events, and situations. a. Self-select text to respond and develop innovative perspectives. b. Establish and use criteria to classify, select, and evaluate texts to make informed judgments about the quality of the pieces.

Reading Standards for Informational Text 6–12 UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details		
1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
2. Determine a central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.	2. Determine two or more central ideas in a text and analyze their development over the course of the text; provide an objective summary of the text.	2. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas; provide an objective summary of the text.
3. Analyze in detail how a key individual, event, or idea is introduced, illustrated, and elaborated in a text (e.g., through examples or anecdotes).	3. Analyze the interactions between individuals, events, and ideas in a text (e.g., how ideas influence individuals or events, or how individuals influence ideas or events).	3. Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories).
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
5. Analyze how a particular sentence, paragraph, chapter, or section fits into the overall structure of a text and contributes to the development of the ideas.	5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas.	5. Analyze in detail the structure of a specific paragraph in a text, including the role of particular sentences in developing and refining a key concept.
6. Determine an author's point of view or purpose in a text and explain how it is conveyed in the text.	6. Determine an author's point of view or purpose in a text and analyze how the author distinguishes his or her position from that of others.	6. Determine an author's point of view or purpose in a text and analyze how the author acknowledges and responds to conflicting evidence or viewpoints.

Reading Standards for Informational Text 6–12 – cont. UNPACKED

Integration of Knowledge and Ideas		
7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue.	7. Compare and contrast a text to an audio, video, or multimedia version of the text, analyzing each medium's portrayal of the subject (e.g., how the delivery of a speech affects the impact of the words).	7. Evaluate the advantages and disadvantages of using different mediums (e.g., print or digital text, video, multimedia) to present a particular topic or idea.
8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not.	8. Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims.	8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced.
9. Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a biography on the same person). a. Use their experience and their knowledge of language and logic, as well as culture, to think analytically, address problems creatively, and advocate persuasively.	9. Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts. a. Use their experience and their knowledge of language and logic, as well as culture, to think analytically, address problems creatively, and advocate persuasively.	9. Analyze a case in which two or more texts provide conflicting information on the same topic and identify where the texts disagree on matters of fact or interpretation. a. Use their experience and their knowledge of language and logic, as well as culture, to think analytically, address problems creatively, and advocate persuasively.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction at the high end of the grades 6–8 text complexity band independently and proficiently.

Reading Standards for Informational Text 6–12 – cont. UNPACKED

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grade 9-10 students:	Grade 11-12 students:
Key Ideas and Details	
1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text. a. Develop factual, interpretive, and evaluative questions for further exploration of the topic(s).	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain. a. Develop factual, interpretive, and evaluative questions for further exploration of the topic(s).
2. Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.	2. Determine two or more central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to provide a complex analysis; provide an objective summary of the text.
3. Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.	3. Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
Craft and Structure	
4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language of a court opinion differs from that of a newspaper).	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
5. Analyze in detail how an author's ideas or claims are developed and refined by particular sentences, paragraphs, or larger portions of a text (e.g., a section or chapter).	5. Analyze and evaluate the effectiveness of the structure an author uses in his or her exposition or argument, including whether the structure makes points clear, convincing, and engaging.
6. Determine an author's point of view or purpose in a text and analyze how an author uses rhetoric to advance that point of view or purpose.	6. Determine an author's point of view or purpose in a text in which the rhetoric is particularly effective, analyzing how style and content contribute to the power, persuasiveness, or beauty of the text.

Reading Standards for Informational Text 6–12 – cont. UNPACKED

Grade 9-10 students:	Grade 11-12 students:
Integration of Knowledge and Ideas	
7. Analyze various accounts of a subject told in different mediums (e.g., a person's life story in both print and multimedia), determining which details are emphasized in each account.	7. Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning.	8. Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning (e.g., in U.S. Supreme Court majority opinions and dissents) and the premises, purposes, and arguments in works of public advocacy (e.g., The Federalist, presidential addresses).
9. Analyze seminal U.S. documents of historical and literary significance (e.g., Washington's Farewell Address, the Gettysburg Address, Roosevelt's Four Freedoms speech, King's "Letter from Birmingham Jail"), including how they address related themes and concepts. a. Read, annotate, and analyze informational texts on topics related to diverse and nontraditional cultures and viewpoints.	9. Analyze seventeenth-, eighteenth-, and nineteenth-century foundational U.S. documents of historical and literary significance (including The Declaration of Independence, the Preamble to the Constitution, the Bill of Rights, and Lincoln's Second Inaugural Address) for their themes, purposes, and rhetorical features. a. Read, annotate, and analyze informational texts on topics related to diverse and non-traditional cultures and viewpoints.
Range of Reading and Level of Text Complexity	
10. By the end of grade 9, read and comprehend literary nonfiction in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 10, read and comprehend literary nonfiction at the high end of the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 11, read and comprehend literary nonfiction in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literary nonfiction at the high end of the grades 11–CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes v

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

Responding to Literature

11. Develop personal, cultural, textual, and thematic connections within and across genres as they respond to texts through written, digital, and oral presentations, employing a variety of media and genres.

Writing Standards 6–12 UNPACKED

The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes		
1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s) and organize the reasons and evidence clearly. b. Support claim(s) with clear reasons and relevant evidence, using credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to clarify the relationships among claim(s) and reasons. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from the argument presented.	1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s), acknowledge alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s), acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument

		presented.

Writing Standards 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes		
2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.

Writing Standards 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes		
3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. d. Use precise words and phrases, relevant descriptive details, and sensory language to convey experiences and events. e. Provide a conclusion that follows from the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. d. Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. e. Provide a conclusion that follows from and reflects on the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, description, and reflection, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence, signal shifts from one time frame or setting to another, and show the relationships among experiences and events. d. Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. e. Provide a conclusion that follows from and reflects on the narrated experiences or events.

Writing Standards 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) a. Produce text (print or nonprint) that explores a variety of cultures and perspectives.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) a. Produce text (print or nonprint) that explores a variety of cultures and perspectives.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.) a. Produce text (print or nonprint) that explores a variety of cultures and perspectives.
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. (Editing for conventions should demonstrate command of Language standards 1–3).	5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. (Editing for conventions should demonstrate command of Language standards 1–3).	5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. (Editing for conventions should demonstrate command of Language standards 1–3).
6. Use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of thee pages in a single sitting.	6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources.	6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others.
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question, drawing on several sources and refocusing the inquiry when appropriate.	7. Conduct short research projects to answer a question, drawing on several sources and generating additional related, focused questions for further research and investigation.	7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.
8. Gather relevant information from multiple print and digital sources; assess the credibility of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and providing basic bibliographic information for sources.	8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.

Writing Standards 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Research to Build and Present Knowledge		
9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply grade 6 Reading standards to literature (e.g., "Compare and contrast texts in different forms or genres [e.g., stories and poems; historical novels and fantasy stories] in terms of their approaches to similar themes and topics"). b. Apply grade 6 Reading standards to literary nonfiction (e.g., "Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply grade 7 Reading standards to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). b. Apply grade 7 Reading standards to literary nonfiction (e.g. "Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply grade 8 Reading standards to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). b. Apply grade 8 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced").
Range of Writing		
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.
Responding to Literature		
11. Create and present a text or art work in response to a literary work a. Develop a perspective or theme supported by relevant details. b. Recognize and illustrate social, historical, and cultural features in the presentation of literary texts. c. Create poetry, stories, plays, and other literary forms (e.g. videos, art work).	11. Create a presentation, art work, or text in response to a literary work with a commentary that identifies connections. a. Make deliberate, personal, cultural, textual, and thematic connections across genres. b. Create poetry, stories, plays and other literary forms (e.g. videos, art work).	11. Create a presentation, art work, or text in response to a literary work with a commentary that identifies connections and explains divergences from the original. a. Make well-supported personal, cultural, textual, and thematic connections across genres. b. Create poetry, stories, plays, and other literary forms (e.g. videos, art work).

Writing Standards 6–12 – cont.

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grade 9-10 Students:	Grade 11-12 students:
Text Types and Purposes	
1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. Explore and inquire into areas of interest to formulate an argument. a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. Explore and inquire into areas of interest to formulate an argument. a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from and supports the argument presented.

Writing Standards 6–12 – cont. UNPACKED

Grade 9-10 Students:	Grade 11-12 students:
Text Types and Purposes	
2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. a. Introduce a topic; organize complex ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. a. Introduce a topic; organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use appropriate and varied transitions and syntax to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).

Writing Standards 6–12 – cont. UNPACKED

Grade 9-10 Students:	Grade 11-12 students:
Text Types and Purposes	
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences. a. Engage and orient the reader by setting out a problem, situation, or observation, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events. b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters. c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole. d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters. e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative. f. Adapt voice, awareness of audience, and use of language to accommodate a variety of cultural contexts.	3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences. a. Engage and orient the reader by setting out a problem, situation, or observation and its significance, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events. b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters. c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole and build toward a particular tone and outcome (e.g., a sense of mystery, suspense, growth, or resolution). d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters. e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative. f. Adapt voice, awareness of audience, and use of language to accommodate a variety of cultural contexts.
Production and Distribution of Writing	
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 9–10.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 11–12.
6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.

Writing Standards 6–12 – cont. UNPACKED

Grade 9-10 Students:	Grade 11-12 students:
Research to Build and Present Knowledge	
7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation. a. Explore topics dealing with different cultures and world viewpoints.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation. a. Explore topics dealing with different cultures and world viewpoints. of language to accommodate a variety of cultural contexts.
8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply grades 9–10 Reading standards to literature (e.g., "Analyze how an author draws on and transforms source material in a specific work [e.g., how Shakespeare treats a theme or topic from Ovid or the Bible or how a later author draws on a play by Shakespeare]"). b. Apply grades 9–10 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply grades 11–12 Reading standards to literature (e.g., "Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics"). b. Apply grades 11–12 Reading standards to literary nonfiction (e.g., "Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning [e.g., in U.S. Supreme Court Case majority opinions and dissents] and the premises, purposes, and arguments in works of public advocacy [e.g., The Federalist, presidential addresses]").
Range of Writing	
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

Writing Standards 6–12 – cont. UNPACKED

Grade 9-10 Students:	Grade 11-12 students:
Responding to Literature	
11. Create literary texts that demonstrate knowledge and understanding of a wide variety of texts of recognized literary merit. a. Engage in a wide range of prewriting experiences, such as using a variety of visual representations, to express personal, social, and cultural connections and insights. b. Identify, analyze, and use elements and techniques of various genres of literature. c. Develop critical and interpretive texts from more than one perspective, including historical and cultural. d. Create poetry, stories, plays, and other literary forms (e.g. videos, art work).	11. Create interpretive and responsive texts to demonstrate knowledge and a sophisticated understanding of the connections between life and the literary work. a. Engage in using a wide range of prewriting strategies, such as visual representations and the creation of factual and interpretive questions, to express personal, social and cultural connections and insights. b. Identify, analyze, and use elements and techniques of various genres of literature, such as allegory, stream of consciousness, irony, and ambiguity, to affect meaning. c. Develop innovative perspectives on texts, including historical, cultural, sociological, and psychological contexts. d. Create poetry, stories, plays, and other literary forms (e.g. videos, art work).

College and Career Readiness Anchor Standards for Speaking and Listening

The 6-12 standards for speaking and listening define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Speaking and Listening Standards 6–12 UNPACKED

The following standards for 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed. c. Pose and respond to specific questions with elaboration and detail by making comments that contribute to the topic, text, or issue under discussion. d. Review the key ideas expressed and demonstrate understanding of multiple perspectives through reflection and paraphrasing. e. Seek to understand and communicate with individuals from different perspectives and cultural backgrounds.	1. Engage effectively in a range of collaborative discussions (one on-one, in groups, and teacher-led) with diverse partners on grade 7 topics, texts, and issues, building on others' ideas and expressing their own clearly. a. Come to discussions prepared having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. d. Acknowledge new information expressed by others and, when warranted, modify their own views. e. Seek to understand other perspectives and cultures and communicate effectively with audiences or individuals from varied backgrounds.	1. Engage effectively in a range of collaborative discussions (one on-one, in groups, and teacher-led) with diverse partners on grade 8 topics, texts, and issues, building on others' ideas and expressing their own clearly. a. Come to discussions prepared having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions and decision making, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. d. Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented. e. Seek to understand other perspectives and cultures and communicate effectively with audiences or individuals from varied backgrounds.

Speaking and Listening Standards 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration		
2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study. a. Use their experience and their knowledge of language and logic, as well as culture, to think analytically, address problems creatively, and advocate persuasively	2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study. a. Use their experience and their knowledge of language and logic, as well as culture, to think analytically, address problems creatively, and advocate persuasively	2. Analyze the purpose of information presented in diverse media and formats (e.g., visually, quantitatively, orally) and evaluate the motives (e.g., social, commercial, political) behind its presentation. a. Use their experience and their knowledge of language and logic, as well as culture, to think analytically, address problems creatively, and advocate persuasively.
3. Delineate a speaker's argument and specific claims, distinguishing claims that are supported by reasons and evidence from claims that are not.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the evidence.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced.
Presentation of Knowledge and Ideas		
4. Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation.
5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.	5. Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points.	5. Integrate multimedia and visual displays into presentations to clarify information, strengthen claims and evidence, and add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 6 Language standards 1 and 3).	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 7 Language standards 1 and 3).	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 8 Language standards 1 and 3).

Speaking and Listening Standards 6–12 – cont. UNPACKED

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grade 9-10 Students:	Grade 11-12 students:
Comprehension and Collaboration	
1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to set rules for collegial discussions and decision-making (e.g., informal consensus, taking votes on key issues, presentation of alternate views), clear goals and deadlines, and individual roles as needed. c. Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions. d. Respond thoughtfully to diverse perspectives, summarize points of agreement and disagreement, and, when warranted, qualify or justify their own views and understanding and make new connections in light of the evidence and reasoning presented. e. Seek to understand other perspectives and cultures and communicate effectively with audiences or individuals from varied backgrounds	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to promote civil, democratic discussions and decision-making, set clear goals and deadlines, and establish individual roles as needed. c. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives. d. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. e. Seek to understand other perspectives and cultures and communicate effectively with audiences or individuals from varied backgrounds.
2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.	2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any fallacious reasoning or exaggerated or distorted evidence.	3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

Speaking and Listening Standards 6–12 – cont.

Grade 9-10 Students:	Grade 11-12 students:
Presentation of Knowledge and Ideas	
4. Present information, findings, and supporting evidence clearly, concisely, and logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task.	4. . Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.
5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.	5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grades 9–10 Language standards 1 and 3).	6. Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate. (See grades 11–12 Language standards 1 and 3).

College and Career Readiness Anchor Standards for Language

The 6–12 standards for language define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

Language Standards 6–12 UNPACKED

The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades. Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*).

Grade 6 students:	Grade 7 students:	Grade 8 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Ensure that pronouns are in the proper case (subjective, objective, possessive). b. Use intensive pronouns (e.g., myself, ourselves). Recognize and correct inappropriate shifts in pronoun c. number and person.* d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).* e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of phrases and clauses in general and their function in specific sentences. b. Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of verbals (gerunds, participles, infinitives) in general and their function in particular sentences. b. Form and use verbs in the active and passive voice. c. Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. d. Recognize and correct inappropriate shifts in verb voice and mood.*
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.* b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a comma to separate coordinate adjectives (e.g., It was a fascinating, enjoyable movie but not He wore an old[,] green shirt). b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (comma, ellipsis, dash) to indicate a pause or break. b. Use an ellipsis to indicate an omission. c. Spell correctly.

Language Standards 6–12 – cont. UNPACKED

Grade 6 students:	Grade 7 students:	Grade 8 students:
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Vary sentence patterns for meaning, reader/listener interest, and style.* b. Maintain consistency in style and tone.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 6 reading and content, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., audience, auditory, audible). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 7 reading and content, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., belligerent, bellicose, rebel). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple meaning words or phrases based on grade 8 reading and content, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., precede, recede, secede). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).

Language Standards 6–12 – cont.

Kindergartners:	Grade 1 students:	Grade 2 students:
Vocabulary Acquisition and Use		
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., personification) in context. b. Use the relationship between particular words (e.g., cause/effect, part/whole, item/category) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., stingy, scrimping, economical, unwasteful, thrifty).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., literary, biblical, and mythological allusions) in context. b. Use the relationship between particular words (e.g., synonym/antonym, analogy) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., refined, respectful, polite, diplomatic, condescending).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g. verbal irony, puns) in context. b. Use the relationship between particular words to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., bullheaded, willful, firm, persistent, resolute).
6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Standards 6–12 – cont. UNPACKED

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grade 9-10 students:	Grade 11-12 students:
Conventions of Standard English	
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use parallel structure.* b. Use various types of phrases (noun, verb, adjectival, adverbial, participial, prepositional, absolute) and clauses (independent, dependent; noun, relative, adverbial) to convey specific meanings and add variety and interest to writing or presentations.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Apply the understanding that usage is a matter of convention, can change over time, and is sometimes contested. b. Resolve issues of complex or contested usage, consulting references (e.g., Merriam Webster's Dictionary of English Usage, Garner's Modern American Usage) as needed.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a semicolon (and perhaps a conjunctive adverb) to link two or more closely related independent clauses. b. Use a colon to introduce a list or quotation. c. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Observe hyphenation conventions. b. Spell correctly.
Knowledge of Language	
3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening. a. Write and edit work so that it conforms to the guidelines in a style manual (e.g., MLA Handbook, Turabian's Manual for Writers) appropriate for the discipline and writing type.	3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening. a. Vary syntax for effect, consulting references (e.g., Tufte's Artful Sentences) for guidance as needed; apply an understanding of syntax to the study of complex texts when reading.

Language Standards 6–12 – cont. UNPACKED

Grade 9-10 students:	Grade 11-12 students:
Vocabulary Acquisition and Use	
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 9–10 reading and content, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., analyze, analysis, analytical; advocate, advocacy). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, or its etymology. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., conceive, conception, conceivable). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, its etymology, or its standard usage. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., euphemism, oxymoron) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., hyperbole, paradox) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.
6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary y knowledge when considering a word or phrase important to comprehension or expression.

Standard 10: Range, Quality, and Complexity of Student Reading 6–12

Measuring Text Complexity: Three Factors

- Qualitative evaluation of the text: Levels of meaning, structure, language conventionality and clarity, and knowledge demands
- Quantitative evaluation of the text: Readability measures and other scores of text complexity
- Matching reader to text and task: Reader variables (such as motivation, knowledge, and experiences) and task variables (such as purpose and the complexity generated by the task assigned and the questions posed)

Range of Text Types for 6–12

Students in 6–12 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods. The following represent a sampling of types and suggestions for types of texts. Note that these are not mandated text but are illustrative of types of text to be utilized. Teachers should use their professional judgment and best practices to choose texts in each genre.

Literature			Informational Text
Stories	**Dramas**	**Poetry**	**Literary Nonfiction and Historical, Scientific, and Technical Texts**
Includes the subgenres of adventure stories, historical fiction, mysteries, myths, science fiction, realistic fiction, allegories, parodies, satire, and graphic novels.	Includes one-act and multi-act plays, both in written form and on film.	Includes the subgenres of narrative poems, lyrical poems, free verse poems, sonnets, odes, ballads, and epics.	Includes the subgenres of exposition, argument, and functional text in the form of personal essays, speeches, opinion pieces, essays about art or literature, biographies, memoirs, journalism, and historical, scientific, technical, or economic accounts (including digital sources) written for a broad audience.

Texts Illustrating the Complexity, Quality, and Range of Student Reading 6–12 (Examples)

Grades	Literature: Stories, Drama, Poetry	Informational Texts: Literary Nonfiction and Historical, Scientific, and Technical Texts
6-8	▪ *Little Women* by Louisa May Alcott (1869) ▪ *The Adventures of Tom Sawyer* by Mark Twain (1876) ▪ "The Road Not Taken" by Robert Frost (1915) ▪ *The Dark Is Rising* by Susan Cooper (1973) ▪ *Dragonwings* by Laurence Yep (1975) ▪ *Roll of Thunder, Hear My Cry* by Mildred Taylor (1976)	▪ "Letter on Thomas Jefferson" by John Adams (1776) ▪ *Narrative of the Life of Frederick Douglass, an American Slave* by Frederick Douglass (1845) ▪ "Blood, Toil, Tears and Sweat: Address to Parliament on May 13th, 1940" by Winston Churchill (1940) ▪ *Harriet Tubman: Conductor on the Underground Railroad* by Ann Petry (1955) ▪ *Travels with Charley: In Search of America* by John Steinbeck (1962)
9-10	▪ *The Tragedy of Macbeth* by William Shakespeare (1592) ▪ "Ozymandias" by Percy Bysshe Shelley (1817) ▪ "The Raven" by Edgar Allen Poe (1845) ▪ "The Gift of the Magi" by O. Henry (1906) ▪ *The Grapes of Wrath* by John Steinbeck (1939) ▪ *Fahrenheit 451* by Ray Bradbury (1953) ▪ *The Killer Angels* by Michael Shaara (1975)	▪ "Speech to the Second Virginia Convention" by Patrick Henry (1775) ▪ "Farewell Address" by George Washington (1796) ▪ "Gettysburg Address" by Abraham Lincoln (1863) ▪ "State of the Union Address" by Franklin Delano Roosevelt (1941) ▪ "Letter from Birmingham Jail" by Martin Luther King, Jr. (1964) ▪ "Hope, Despair and Memory" by Elie Wiesel (1997)
11-CCR	▪ "Ode on a Grecian Urn" by John Keats (1820) ▪ *Jane Eyre* by Charlotte Brontë (1848) ▪ "Because I Could Not Stop for Death" by Emily Dickinson (1890) ▪ *The Great Gatsby* by F. Scott Fitzgerald (1925) ▪ *Their Eyes Were Watching God* by Zora Neale Hurston (1937) ▪ *A Raisin in the Sun* by Lorraine Hansberry (1959) ▪ *The Namesake* by Jhumpa Lahiri (2003)	▪ *Common Sense* by Thomas Paine (1776) ▪ *Walden* by Henry David Thoreau (1854) ▪ "Society and Solitude" by Ralph Waldo Emerson (1857) ▪ "The Fallacy of Success" by G. K. Chesterton (1909) ▪ *Black Boy* by Richard Wright (1945) ▪ "Politics and the English Language" by George Orwell (1946) ▪ "Take the Tortillas Out of Your Poetry" by Rudolfo Anaya (1995)

Implementing Grades 6-12 Altmar Parish Williamstown Central School District Common Core Learning Standards
<u>Reading</u> and Writing Curriculum Units

ELA Period 45 minutes		Instructional Materials/Resources
Reading Unit:		
Opening (3 minutes)	Review of daily student learning objectives	APW CCLS Reading Curriculum Units supported by current APW textbooks, trade books and non-fiction text and delivered utilizing research-based effective instructional strategies. *At this time, teachers should utilize texts that currently exist in their buildings with guidance from Literacy Coaches according to text complexity recommendations and guidelines. If they have access to texts recommended in the APW CCLS Reading units, those texts should be prioritized over other options.
Whole-Group Instruction (15-20 minutes)	Daily Word Study/Vocabulary Read-Aloud/Think-Aloud Demonstration of Comprehension Strategies	
Individual, Partner, and Small Group Period (15-20 minutes)	Structured independent or paired reading Guided and independent practice in making claims about text and supporting claims with textual evidence Text discussion/Accountable Talk Response writing	
Closing (5 minutes)	Student sharing of key points Reflection on daily student learning objectives	

Suggested Instructional Routines – English Language Arts
Implementing Grades 6-12 Altmar Parish Williamstown Central School District Common Core Learning Standards

ELA Period 45 minutes		Instructional Materials/Resources
Writing Unit		
Opening (5 minutes)	Review of daily student learning objectives	APW CCLS Writing Curriculum Units supported by current APW resources and delivered utilizing research-based effective instructional strategies.
Writing Mini-lesson (10-15 minutes)	Teacher models new writing strategy Students practice strategy with guidance and feedback from the teacher.	
Independent Writing (20-25 minutes)	Teacher guides students through the phases of the writing process. Students work independently while teacher offers support to individuals or small groups based on student progress according to corresponding rubric. This independent writing time is critical to the level of stamina and skill demanded by the Common Core Learning Standards.	
Closing (5 minutes)	Student sharing of written work/key learning Reflection on daily student learning objectives.	

ELA SAMPLE PLANS

Successful readers develop good reading strategies that help them read fluently and to read for meaning. In kindergarten through fifth grade classrooms, effective habits of literacy begin to be developed through the implementation of the NYS Common Core Curriculum Units. This guidebook serves to communicate how these pieces can work together to help students develop habits of literacy that will support their learning as they engage in a rigorous instructional program that is more closely aligned to the CCLS.

ALIGNED TO COMMON CORE

Grade 6 Sample Lesson Plan

Instructional Framework ELA

Title/Topic/Theme:

- What Really Happened: The Mystery of King Tutankhamen's Death?

Skill/Content:

- Students will explore what historical and recent research has revealed about King Tut in an effort to solve the mystery of how he died. They will then learn more about how new technology is helping scientists clarify the circumstances surrounding Tut's death.

Common Core State Standards:

- RI.6.1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.
- RI.6.8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not.
- SL.6.1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.
- SL.6.4. Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation.

Essential Question(s)/Guiding Question(s):

- What are some of the theories used to explain King Tutankhamen's mysterious death?
- How has new technology aided in the study of King Tut?

Objective/Result:

- Determine what is known about King Tut's death.
- Investigate new technology aiding in the study of King Tut.

Bridge/Connections:

- Ask students to share what they know about King Tut and how they think he died.

Materials/Resources:

- News articles: -*King Tut Not Murdered Violently, CT Scans Show* http://news.nationalgeographic.com/news/2005/03/0308_050308_kingtutmurder.html
- *Who Killed King Tut?* http://www.time.com/time/magazine/article/0,9171,349108-1,00.html

Lesson/Process/Procedures:

- Assign half of your students to read one of the articles and half of them to read the other. Each article presents a competing theory about King Tut's death, so do not let students know that they are reading different articles.
- Tell the students to take notes on the articles as they read. When they have finished reading, group the students according to the article they have read (again, without letting students know that they read different articles) and have them discuss what they learned. Be sure to position

the groups far enough away from each other so that neither group can hear what the other is discussing. Have each group discuss the following questions:

- How do you think King Tut died?
- What evidence supports your theory?
- What questions remain unanswered, if any?

- Once they reach a consensus, groups should elect one person to present their theory on King Tut's death. Have each group representative state the group's theory about Tut's death and briefly outline the supporting evidence. Once both groups have presented their theories, it should become obvious to students that they were assigned to read different articles. Tell the students that half of them read an article from 2002 and half of them read an article from 2005.

Work Time/Embedded Performances:

- Have students write down the theory they believe best explains Tut's death. Then have them list the unanswered questions that still remain. What information would they need to answer these questions? Have students design a cutting edge technology that they think would provide them with the necessary information to answer the remaining questions. Instruct students to write brief summaries of their technology, explaining how it works, which questions it will answer, and what they believe the answers will be.

Closing/Wrap-Up:

- Invite students to share their writing with the class.

Evaluation/Assessment:

- Evaluate student participation in the group discussion, and students written responses.

Homework/Extensions/Enrichment:

- Have students create a timeline of King Tut research from 1923 through the future. They should focus on specific discoveries and technologies used to prove or disprove theories about his death. To fill in the "future" portion of the timeline, they should consider what other information we may obtain as technology evolves. (Examples of new or evolving technology could include cloning, DNA analysis, MRI scans, etc.).

Name_____

Theories About King Tut's Death	*Methods Used To Learn About Tut*

Name_____

How do you think King Tut died?
What evidence supports your theory?
What questions remain unanswered, if any?

Grade 7 Sample Plan

Title/Topic/Theme:

- Real- Life Bully Proofing

Essential Question(s)/Guiding Question(s):

- How do we persuade others using the problem-solution method?

Common Core Learning Standards:

- CCSS.ELA-Literacy.RI.7.5 Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas.
- CCSS.ELA-Literacy.RI.7.9 Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts.
- CCSS.ELA-Literacy.CCRA.W.1 Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
- CCSS.ELA-Literacy.CCRA.W.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Objective/Result:

The student will:

- Conduct a Close Reading of a nonfiction article related to bullying.
- Write a problem-solution letter.

Bridge:

- After reading the entry from Tangerine for Monday, December 20, have students brainstorm a list of bullying incidents discussed in the text.

Materials/Resources:

- Springboard books
- Copies of Tangerine
- Sourcebooks or looseleaf paper

Mini-lesson:

- Form students in to groups of 3 to 4 and have half of the groups read “Bullying in Schools” and half read “Taming Wild Girls”
- Have each group conduct a close read of their assigned text and mark the text by circling examples of bullying and highlighting solutions presented.
- Have students share in a jigsaw format presenting the examples and solutions they found.
- As students record the solutions to bullying, ask them to write about possible solutions to the acts of bullying in Tangerine.

Work Period:

- As a group, complete the RAFT organizer deciding on a role they will assume for the writing of a persuasive letter.
- Explain they will be using the problem solution format to complete the letter.
- Guide students on the organization of an introduction a body, and a conclusion in their business letters.

Summary:

- Have students share letters in their groups to comment on the positive points of persuasion and one suggestion for improvement for each letter shared.

Closing/Wrap-Up:

- Allow students to self-edit, revise and prepare a final version and read several aloud to the class.

Homework/Extensions/Enrichment:

- Writing of the letter can be assigned as homework.

Grade 8 Sample Plan

Title/Topic/Theme:
- Unit 1: Lesson 18

Skill/Content:
- Text Citation
- Determining Importance
- Analyzing Text

Common Core Learning Standards:
- CCSS.ELA-Literacy.RI.8.1 Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
- CCSS.ELA-Literacy.RI.8.8 Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced.
- CCSS.ELA-Literacy.W.8.1 Write arguments to support claims with clear reasons and relevant evidence.

Essential Question(s)/Guiding Question(s):
- What defines a hero?

Objective/Result:
The student will:
- explore the concept of heroism in real life and nonfiction text
- practice text citations using quotations
- create a well developed paragraph

Bridge/Connections:
- Students have been reading, writing and watching clips about heroes throughout the unit. This activity will allow students to explore the idea of an everyday hero.
- Have students write a one minute essay on what they think the qualities of a hero are.
- Have students share.

Materials/Resources:
- Springboard texts
- Student Sourcebooks

Lesson/Process/Procedures:
- Students will conduct a close read the four paragraphs in Activity 1.14.
- After students have read each piece, they will identify the most important quote from each section that defines Ana as a hero. Students will write these down using appropriate text citation. Teachers may need to review text citation from previous lessons in Unit 1 prior to completion of this activity.
- After students have selected a quote from each piece, the teacher will divide the class up into groups of 3-4 students. Give students a chance to share their quotes with each

other. Then, the students must decide on one overall quote that best fits Ana. Allow students ample time to engage in conversation surrounding the selected quotes.

- In their student sourcebooks, students can write about why they selected the quote and why the quote fits Ana as an everyday hero.
    - *Note: Questions in Activity 1.14 should be used as a guide for the writing process.

Work Time/Embedded Performances:

- Completion of Sourcebook entry paragraph discussing quotation from text.
- Text citation of quotes from each piece describing Ana as a hero.

Closing/Wrap-Up:

- For closing, groups can share the quote that they selected and give a rationale of why they selected that quote. If time permits, students can share portions of their writing pieces.

Evaluation/Assessment:

- Activity assessments are based on observation, participation and completion of the tasks described above.

Homework/Extensions/Enrichment:

- If needed, students can complete the writing piece for homework and bring in the next day to share.
- Additionally, students can complete their own quickwrite about an everyday hero in their life.

Name

An Everyday Hero

Piece Number	Unknown Word	Meaning

Name _____ Quotes

Directions: Choose ***one quote*** that best describes Ana as a hero. Explain why you choose that quote using at least one detail from the story you have read.

Piece One –

"She is an inspiration to us all, for she is a real hero."

"...I know she is afraid, she fights…every moment of every day, and she does it with a smile."

Piece Two –

"Day after day, my mother gets up with a smile and still has the strength to be a mother to me."

"I know I'm young, but I know I will never catch up to my mother's faith."

Piece Three –

"A hero can be a savior, a fighter, or a mother changing her child's diaper."

"Even though chances were slim, she would keep on fighting 'til the very end."

Why did you choose that quote? Provide evidence from the text as it describes Ana as a hero.

Grade 9 Sample Plan

ELA Grade 9 Introductory Unit

Supplemental Lesson – Building Community

Title/Topic /Theme: Building Community

Time-Frame: 4-5 days

Skill / Content:

- Students will analyze a painting, (Graphic Organizer follows) making observations about its content;
- Students will use their understanding of community to analyze a painting
- Students use these observations to draw conclusions about what is important to the painter (Henry Ossawa Tanner);
- Students will explore the concept of conflict, in text, as it can be a defining moment in a person's life
- Students will demonstrate their understanding of how relationships support "personhood" *(Note: Personhood is an appreciation that every person is unique: personalities, experiences, knowledge, preferences and life history all make up what is called personhood. It is all the things that make people different from each other. Helping someone to preserve their personhood will have a dramatic effect on their sense of well-being.)*
- Students will use guiding questions to analyze a complex text ("The Boy Who Painted Christ Black," by John Henrik Clarke)
- Students will build vocabulary using a personal thesaurus format developed by the teacher (See "Personal Thesaurus Template" and Personal Thesaurus directions" attachments)
- Students will review Short Story Elements (Short Story Analysis worksheet)
- Students will activate schema to connect their own defining moment to complete the Baseline Narrative (Embedded Assessment 1)

Common Core Learning Standards

- CCSS.ELA-Literacy.RL.9-10.1 Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.
- CCSS.ELA-Literacy.RL.9-10.2 Determine a theme or central idea of a text and analyze in detail its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.
- CCSS.ELA-Literacy.RL.9-10.4 Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language evokes a sense of time and place; how it sets a formal or informal tone)
- CCSS.ELA-Literacy.RL.9-10.10 By the end of grade 9, read and comprehend literature, including stories, dramas, and poems, in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range

Cultural Learning Standards: (RTC, 2007)

- Students understand that they are central to the learning process.
- Students know and feel that they belong – they are safe, cared for, and acknowledged in classrooms as individuals and as members of diverse cultures and groups.
- Students know that schools are a center of community life.
- Students know and expect that families and community play a significant role in the educational process.

Essential Questions:

- How does an individual shine when a community supports him/her?
- Why do we need each other?
- How do moments of conflict empower us?
- How can we demystify words?

Objective / Result:

- Students will begin to consider the importance of COMMUNITY as necessary component of personhood.
- Students will know that they belong and are affirmed as being part of a diverse cultural group.
- Students will understand complex text through inquiry.

Bridge / Connections:

- Observing Henry Ossawa Tanner's painting "The Banjo Lesson" (1893, oil on canvas), students will complete the Picture Analysis Worksheet (Side One)- (Project painting on SmartBoard, or print—in color—on overhead sheet, or make a color copy, or as a last resort, make black and white copies)

Materials / Resources:

- Double Sided Picture Analysis Worksheet/Short Story Analysis Sheet
- Image of "The Banjo Lesson" to project
- Personal Thesaurus 'binder' (See Personal Thesaurus Template and Personal Thesaurus Directions worksheets, or teacher to develop process for recording vocabulary) – NOTE: Teacher will need to develop a mini-lesson to introduce personal thesaurus, or other means of recording vocabulary throughout the year.
- Copies of "The Boy Who Painted Christ Black" Student Text
- "The Boy Who Painted Christ Black" Teacher Text with Inquiry questions and highlighted Vocabulary – plain text and/or hyperlinked text can be uploaded to SmartBoard if available
- Teacher Notes for Story Analysis Sheet
- Markers for highlighting the text
- Dictionaries / Thesaurus' hardcopy or online

Learning Process / Procedures:

- **Day 1-2**
  - Students will share their analysis of the painting, "The Banjo Lesson"
  - Through inquiry, teacher should guide the students to make inferences about the relationship depicted in the painting, and inferences about the painter himself. (What can you tell me about the relationship between the elder and the young boy? How do you know (student should provide evidence from painting to support answer)? What do you know about the person created this painting?
  - Hand out Text
  - Mini-Lesson: Understanding CONNOTATION versus DENOTATION
  - Explain to students that teacher will ask guiding questions about the text AND throughout the text, and students will be expected to highlight the evidence in the text to support their answers to the guiding questions—Model the first two questions (this is very similar to a Socratic Seminar)
  - Prior to reading the text, also instruct students to highlight vocabulary that is new and unfamiliar to them, or familiar vocabulary that is used in an unfamiliar way.
  - Begin reading text paying attention to Guiding Questions and Notes on the TEACHER TEXT for "The Boy Who Painted Christ Black"
- **Day 2-3**
  - Continue reading the text and directing students with guiding questions
  - Upon completing the text, ask students to work together to complete the Short Story Analysis worksheet (Side Two)
  - Closing for Day 2 –have students Think Pair Share to develop a thematic statement about 'relationship' (in the context of the story)
- **Day 3**

  Mini Lessons to support student learning

  1. Word Choice: Connotation vs Denotation – in terms of developing Personal Thesaurus
  2. Personal Thesaurus (or teacher format for recording vocabulary throughout the year)
     - Model the Personal Thesaurus
     - Jigsaw sections of the text and group students to add to their Personal Thesaurus (Students can use the teacher text with hyperlinked vocabulary)
- **Day 4-5**
  - Students share their vocabulary for larger group to add to personal thesaurus

Work Time / Embedded Assessment:

- Students analyze picture using worksheet
- Students mark text for information
- Students participate in discussion by finding evidence in text to support answers to teacher's guided questions
- Students complete short story analysis worksheet
- Students add to their personal thesaurus

Closing Wrap –up:

- Based on Jigsaw activity for the Personal Thesaurus, answer the Essential Question Why do we need each other?

Evaluation / Assessment:

- Picture Analysis Worksheet/Story Notetaking Organizer
- Participation in Inquiry of text
- Marking of the text
- Thematic Statement about Relationship
- Personal Thesaurus

Homework / Extension:

- Day 1: Students to research Henry Ossawa Tanner, and write a summary of their findings
- Day 2: Students journal about what they feel they are good at
- Day 3: Students brainstorm events or people that have changed their life
- Day 4-5: Students choose minimum of five words from their personal thesaurus to write a minimum of one paragraph (5 sent) about important relationships in their lives

Grade 10 Sample Plan

Title/Topic/Theme:
- Unit 2: Lesson 3

Skill/Content:
- To analyze tone and diction to track changes in narrative perspective
- To analyze tone and diction and find textual evidence to support a claim

Common Core Learning Standards:
- CCSS.ELA-Literacy.RL.9-10.1 Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.
- CCSS.ELA-Literacy.RL.9-10.2 Determine a theme or central idea of a text and analyze in detail its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text
- CCSS.ELA-Literacy.W.9-10.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

Essential Question(s)/Guiding Question(s):
- How does an author's tone reflect their changing attitudes in a text?

Objective/Result:

The student will:
- Conduct a close read of a narrative nonfiction text.
- Analyze how changing perspectives can be identified in a text and reflected in tone
- Create a well-developed paragraph analyzing author's word choice to communicate a change in perspective over time

Bridge/Connections:
- To activate prior knowledge, instruct students to complete the quickwrite about a Thanksgiving they have celebrated or have seen celebrated on television or in books or movies.

Materials/Resources:
- Springboard texts
- Student Sourcebooks

Lesson/Process/Procedures:
- Introduce Thanksgiving: A personal history to students and explain that this is an author reflecting on her life through the lens of a specific holiday. You may ask students to consider how their views on their birthdays have changed as they have grown older.
- Students will conduct a close read of Thanksgiving: A Personal History Have them read once to get the Gist. The second read should be conducted so they can identify the shifts or changes in perspective from childhood, to adolescence, to adulthood.
- Complete the Words/ Phrases/ and Tone Graphic organizer with students by drawing on the board or smartboard.
- Work together as a class to identify how the author feels at different points in her life.
- Have students complete the graphic organizer on the bottom of page 66.

- Have students think-pair-share to discuss and determine the time periods in the author's life and the author's attitude at different points in her life.
- Ask students to construct a paragraph response to the questions at the bottom of page 66.

Work Time/Embedded Performances/ Assessment:

- Completion of Graphic Organizer
- Completion and quality of paragraph response

Closing/Wrap-Up:

- For closing, students can share the paragraph they constructed and reflect on the strategies used to identify shifts in tone in the passage.

Evaluation/Assessment:

- Activity assessments are based on observation, participation and completion of the tasks described above.

Homework/Extensions/Enrichment:

- If needed, students can complete the writing piece for homework and bring in the next day to share.

Grade 11 Sample Plan

Title/Topic/Theme:
- Unit 1: Lesson 13

Skill/Content:
- To apply the elements of argument in response to a literary nonfiction text.

Common Core Learning Standards:
- CCSS.ELA-Literacy.RI.11-12.1 Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
- CCSS.ELA-Literacy.W.11-12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. And corresponding subsections of this standard.

Essential Question(s)/Guiding Question(s):
- How can we use the problem/solution framework for persuasion?

Objective/Result:
The student will:
- Complete a double entry journal while reading tan excerpt of an essay.
- Students will identify five important passages and respond to the passages.
- Create a well-developed response to the passage in the form of a persuasive letter citing quotations from the passage.

Bridge/Connections:
- Review strategies for a close read with students identifying how to read once to get the GIST of a passage, and once to find significant or in this case emotionally-charged quotations or passages.

Materials/Resources:
- Springboard texts
- Student Sourcebooks

Lesson/Process/Procedures:
- Instruct students they will conduct a close read of Nickel and Dimed first to find the main points, and secondly to complete their double entry journal.
- Instruct students to complete a double entry journal while reading the excerpt from *Nickel and Dimed*.
- After students have read the text move students in to discussion groups of up to four students to engage in a group discussion of the text.
- Ask groups to share out the issues discussed from texts.
- Review the elements of hook, claim, support, concessions and refutations and call to action.

Work Time/Embedded Performances/ Assessment:

- Students will compose a persuasive letter to either a news editorial or a maid service company offering suggestions for improving conditions for workers.

Closing/Wrap-Up:

- For closing, students should share their persuasive letters with the class and review the components of a persuasive letter particular to hook, claim, support, and refutations, etc.

Evaluation/Assessment:

- Activity assessments are based on observation, participation and completion of the tasks described above.

Homework/Extensions/Enrichment:

- If needed, students can complete the writing piece for homework and bring in the next day to share.

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Grade 12 Sample Plan

Title/Topic/Theme:
- Unit 1: Lesson 17: Reading with a Cultural Criticism Lens

Skill/Content:
- To analyze the elements of an essay using the lens of Cultural Criticism.

Common Core Learning Standards:
- CCSS.ELA-Literacy.RI.11-12.1 Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
- CCSS.ELA-Literacy.RI.11-12.2 Determine two or more central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to provide a complex analysis; provide an objective summary of the text.
- CCSS.ELA-Literacy.RI.11-12.3 Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
- CCSS.ELA-Literacy.RI.11-12.6 Determine an author's point of view or purpose in a text in which the rhetoric is particularly effective, analyzing how style and content contribute to the power, persuasiveness or beauty of the text.

Essential Question(s)/Guiding Question(s):
- How can we use a specific lens to further recognize the relevance and beauty of a text?

Objective/Result:
The student will:
- Complete a close read of "Shooting an Elephant"
- Explore the concepts and structures of a reflective essay
- Create and revise a reflective essay in response to the text
- Generate leveled questions to participate in a Socratic Seminar

Bridge/Connections:
- Ask students to present briefly on the research they have conducted pertaining to imperialism from the previous activity..

Materials/Resources:
- Springboard texts
- Student Sourcebooks

Lesson/Process/Procedures:
- Students should begin by attempting to diffuse the difficult vocabulary of the text by skimming for them and using context clues to replace these words with more familiar terms.
- Instruct students they will conduct a close read of Shooting an Elephant first to find the main points, and secondly to complete their double entry journal.
- Prepare for students the visual display of a reflective essay organizational pattern and review the three main parts, the event, the response, and the reflection.
- Ask students to consider a significant event in their own lives and draft a brief reflective response based on the event, their response, and what they learned.
- Have students complete a think-pair-share with their responses.

- Assist students by showing them how to chunk the text in to smaller parts.
    - The first two paragraphs establish mood, tone, and setting.
    - Paragraphs 3 and 4 introduce the issue or factors leading up to the event
    - Paragraphs 5 and 6 discuss the narrator's feelings towards the situation
    - Paragraphs 8 and 9 discuss the author's positions and internal dialogue
    - Paragraphs 10-12 relay the actual event
    - The remainder of the essay relay the narrator's reflections
- . Have students reflect on the narrator's response, his thoughts at the time, and the theme of the essay.

Work Time/Embedded Performances/ Assessment:

- Have students revisit their essays and revise them now that they have a greater understanding of the genre. Students should also prepare questions to conduct a Socratic Seminar regarding the essay they have read.

Closing/Wrap-Up:

- For closing, students should prepare for the Socratic Seminar applying Cultural Criticism to the essay.

Evaluation/Assessment:

- Activity assessments are based on observation, participation in the Socratic Seminar and completion of the essay described above.

Homework/Extensions/Enrichment:

Core Instructional program

Mathematics

New York State Common Core Standards for Mathematical Practice 6 – 12

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. **Make sense of problems and persevere in solving them.**

   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. . **Reason abstractly and quantitatively**.

   Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and

other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

How to Read the Grade Level Standards

Standards define what students should understand and be able to do.

Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know A should next come to learn B.” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Mathematics - Grade 6: Introduction

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Mathematics - Grade 6: Introduction – cont.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems..

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expression and Equation

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Ratios & Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
    b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
    c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
    d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (*a/b*) ÷ (*c/d*) = *ad/bc*.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
    a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
    b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
    c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
7. Understand ordering and absolute value of rational numbers.
    a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.
    b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –30 C > –70 C to express the fact that –30 C is warmer than –70 C.
    c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
    d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions & Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
    a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – *y*.
    b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
    c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.
3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
7. Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
8. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics & Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
    a. Reporting the number of observations.
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics - Grade 7: Introduction

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems..

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expression and Equation

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about
- two populations. • Investigate chance processes and develop, use, and evaluate probability models..

Ratios & Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.
2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number *n* of items purchased at a constant price *p*, the relationship between the total cost and the number of items can be expressed as $t = pn$.
    d. Explain what a point (*x, y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
    a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
    b. Understand $p + q$ as the number located a distance $|q|$ from *p,* in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
    c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
    d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
    a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
    b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.
    c. Apply properties of operations as strategies to multiply and divide rational numbers.
    d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
3. Solve real-world and mathematical problems involving the four operations with rational numbers.

Expressions & Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, *a* + 0.05*a* = 1.05*a* means that "increase by 5%" is the same as "multiply by 1.05."

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
    a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
    b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions

Geometry 7.G

Draw construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics & Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
    a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
    b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
    b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
    c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Mathematics - Grade 8: Introduction

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

   Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expression and Equation

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities of simultaneous linear equations.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems

Statistics and Probability

- Investigate patterns of association in bivariate data.

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions & Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 108 and the population of the world as 7 times 109, and determine that the world population is more than 20 times larger.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance time equation to determine which of two moving objects has greater speed.
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at *b*.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.
    c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
3. Interpret the equation y = *mx* + *b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

1 Function notation is not required in Grade 8.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics & Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group**.**

Mathematics - High School Number & Quantity: Introduction

Numbers and Number Systems.

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3, … Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (51/3)3 should be 5(1/3)3 = 51 = 5 and that 51/3 should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities.

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g. acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process might be called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Number and Quantity Overview

The Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems

The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

The Real Number System N.RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N.Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N.CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector & Matrix Quantities N.VN

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, $\mathbf{v}$).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
    c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
    b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Mathematics - High School Algebra: Introduction

Expressions.

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities.

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

Mathematics - High School Algebra: Introduction – cont.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b1+b2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling.

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Algebra Overview

Seeing Structure in Expressions
- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve
- Problems.

Arithmetic with Polynomials and Rational Expressions
- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Creating Equations
- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities
- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Seeing Structure in Expressions A.SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.
2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15t$ can be rewritten as $(1.151/12)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
2. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

Arithmetic with Polynomials & Rational Expressions A.APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
5. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument

Creating Equations A.CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning with Equations & Inequalities A.REI

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

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Mathematics - High School Functions: Introduction

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Functions Overview

Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Interpreting Functions F.IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f(x)*.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by *f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)* for *n ≥ 1*.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function *h(n)* gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end
    d. behavior.
    e. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    f. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)*t, y* = (0.97)*t, y* = (1.01)*12t*, *y* = (1.2)*t*/10, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions F.IF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions.

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for $x \neq 1$.
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, & Exponential Models F.LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F.TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for *x*, where *x* is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics - High School Modeling: Introduction

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

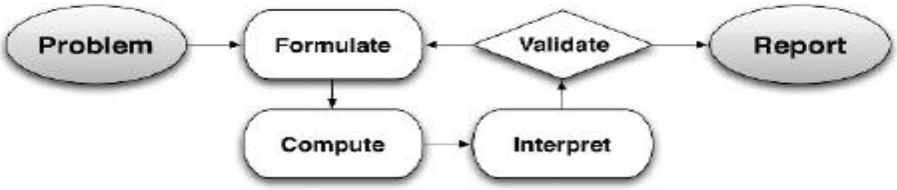
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.

Relating population statistics to individual predictions.
In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

Mathematics - High School Modeling: Introduction – cont.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the 61mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

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Mathematics - High School Geometry: Introduction

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.).

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Mathematics - High School Geometry: Introduction - cont.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations.

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional
- and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Congruence G.CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, & Trigonometry G.SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
    a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G.C

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G.GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula

Geometric Measurement & Dimension G.GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G.MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematics - High School Statistics & Probability: Introduction

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of twoway tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling.

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Interpreting Categorical & Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
    b. Informally assess the fit of a function by plotting and analyzing residuals.
    c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences & Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability & the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P(A and B)/P(B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
7. Apply the Addition Rule, *P(A* or *B*) = *P(A*) + *P(B*) – *P(A* and *B*), and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, *P(A* and *B*) = *P(A)P(B*|*A*) = *P(B)P(A*|*B*), and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
    a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
    b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note on courses & transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

New York State Common Core
Mathematics Curriculum Overview

Grades 6-8

Introduction

This document provides an overview of the academic year for Grades 6 through 8, beginning with a curriculum map and followed by detailed grade level descriptions.

The curriculum map is a chart that shows, at a glance, the sequence of modules comprising each grade of the Grades 6 through 8 curriculum. The map also indicates the approximate number of instructional days designated for each module of each grade. The date approximations are based on an academic calendar beginning on 9/6/12 and ending on 6/26/13 with a testing date approximately mid-late April. Details that elaborate on the curriculum map are found in the grade-level descriptions.

Each grade-level description begins with a list of the five to seven modules that comprise the instruction of that grade. That introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the alignment chart with the grade-level standards.

The "Summary of Year" portion of each grade level includes four pieces of information:

- The critical instructional areas for the grade, as described in the Common Core Learning Standards[1](CCLS)
- The Key Areas of Focus[2] for the grade
- The Required Fluencies for the grade
- The CCLS Major Emphasis Clusters[3] for the grade

The "Rationale for Module Sequence" portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The alignment chart for each grade lists the CCLS that are addressed in each module of the grade. Note that when a cluster is referred to without a footnote, it is taught in its entirety. There are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module, so that a strong foundation can be built over time. Note that the standards identified on the Pre-Post Standards[4] document as those which should be taught after the state test in April, have been intentionally aligned with the final modules of those grades.

[1]EngageNY: http://www.p12.nysed.gov/ciai/common_core_standards/pdfdocs/nysp12cclsmath.pdf
[2]Achievethecore: http://www.achievethecore.org/downloads/E0702_Description_of_the_Common_Core_Shifts.pdf
[3]EngageNY: http://engageny.org/sites/default/files/resource/attachments/nys-math-emphases-k-hs.pdf
[4]NYSED: http://www.p12.nysed.gov/assessment/ei/2013/draft-math-ccls-13.pdf

New York State COMMON CORE MATHEMATICS CURRICULUM **A Story of Ratios Curriculum Overview**

Test Date		Grade 6	Grade 7	Grade 8	
9/6/12	20 days	M1: Ratios and Unit Rates (35 days)	M1: Ratios and Proportional Relationships (30 days)	M1: The Number System and Properties of Exponents (20 days)	20 days
10/10/12	20 days			M2: Congruence (25 days)	20 days
11/8/12	20 days	M2: Arithmetic Operations Including Dividing by a Fraction (25 days)	M2: Rational Numbers (30 days)		
12/11/12	20 days	M3: Rational Numbers (25 days)		M3: Similarity (25 days)	20 days
1/17/13	20 days		M3: Expressions and Equations (35 days)		20 days
2/15/13	20 days	M4: Expressions and Equations (45 days)		M4: Linear Equations (40 days)	20 days
3/22/13	20 days		M4: Percent and Proportional Relationships (25 days)	M5: Examples of Functions from Geometry (15 days)	20 days
4/29/13	20 days	M5: Area, Surface Area, and Volume Problems (25 days)	M5: Statistics and Probability (25 days)	M6: Linear Functions (20 days)	20 days
5/28/13	20 days	M6: Statistics (25 days)	M6: Geometry (35 days)	M7: Introduction to Irrational Numbers Using Geometry (35 days)	20 days
6/26/13	Note that date approximations are based on a first student day of 9/6/12 and last day of 6/26/13.				

Approx. test date for Grades 6-8

Key:	Number	Geometry	Ratios and Proportions	Expressions and Equations	Statistics and Probability	Functions

Sequence of Grade 6 Modules Aligned with the Standards

- Module 1: Ratios and Unit Rates
- Module 2: Arithmetic Operations Including Dividing by a Fraction
- Module 3: Rational Numbers
- Module 4: Expressions and Equations
- Module 5: Area, Surface Area, and Volume Problems
- Module 6: Statistics

A breakdown of each Module is available at the following link: http://engageny.org/resource/grades-6-8-mathematics-curriculum-map

Summary of Year:
Sixth grade mathematics is about (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

CCLS Major Emphasis Clusters
Ratios and Proportional Relationships • Understand ratio concepts and use ratio reasoning to solve problems.
The Number System • Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Apply and extend previous understandings of numbers to the system of rational numbers.
Expressions and Equations • Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.

Key Areas of Focus for Grade 6: Ratios and proportional reasoning; early expressions and equations

Required Fluency: 6.NS.2 Multi-digit division
6.NS.3 Multi-digit decimal operations

Rationale for Module Sequence in Grade 6:
In Module 1, students build on their prior work in measurement and in multiplication and division as they study the concepts and language of ratios and unit rates. They use proportional reasoning to solve problems. In particular, students solve ratio and rate using tape diagrams, tables of equivalent ratios, double number line diagrams, and equations. They plot pairs of values generated from a ratio or rate on the first quadrant of the coordinate plane.

Students expand their understanding of the number system and build their fluency in arithmetic operations in Module 2. Students learned in Grade 5 to divide whole numbers by unit fractions and unit fractions by whole numbers. Now, they apply and extend their understanding of multiplication and division to divide fractions by fractions. The meaning of this operation is connected to real-world problems as students are asked to create and solve fraction division word problems. Students continue (from Fifth Grade) to build fluency with adding, subtracting, multiplying, and dividing multi- digit decimal numbers using the standard algorithms.

Sequence of Grade 6 Modules Aligned with the Standards – cont.

Major themes of Module 3 are to understand rational numbers as points on the number line and to extend previous understandings of numbers to the system of rational numbers, which now include negative numbers. Students extend coordinate axes to represent points in the plane with negative number coordinates and, as part of doing so, see that negative numbers can represent quantities in real-world contexts. They use the number line to order numbers and to understand the absolute value of a number. They begin to solve real-world and mathematical problems by graphing points in all four quadrants, a concept that continues throughout to be used into high school and beyond.

With their sense of number expanded to include negative numbers, in Module 4 students begin formal study of algebraic expressions and equations. Students learn equivalent expressions by continuously relating algebraic expressions back to arithmetic and the properties of arithmetic (commutative, associative, distributive). They write, interpret, and use expressions and equations as they reason about and solve one-variable equations and inequalities and analyze quantitative relationships between two variables.

Module 5 is an opportunity to practice the material learned in Module 4 in the context of geometry; students apply their newly acquired capabilities with expressions and equations to solve for unknowns in area, surface area, and volume problems. They find the area of triangles and other two- dimensional figures and use the formulas to find the volumes of right rectangular prisms with fractional edge lengths. Students use negative numbers in coordinates as they draw lines and polygons in the coordinate plane. They also find the lengths of sides of figures, joining points with the same first coordinate or the same second coordinate and apply these techniques to solve real-world and mathematical problems.

In Module 6, students develop an understanding of statistical variability and apply that understanding as they summarize, describe, and display distributions. In particular, careful attention is given to measures of center and variability.

Sequence of Grade 7 Modules Aligned with the Standards

- Module 1: Ratios and Proportional Relationships
- Module 2: Rational Numbers
- Module 3: Expressions and Equations
- Module 4: Percent and Proportional Relationships
- Module 5: Statistics and Probability
- Module 6: Geometry

A breakdown of each Module is available at the following link: http://engageny.org/resource/grades-6-8-mathematics-curriculum-map

Summary of Year:
Seventh grade mathematics is about (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two-and three-dimensional shapes to solve problems involving
area, surface area, and volume; and (4) drawing inferences about populations based on samples.

CCLS Major Emphasis Clusters
Ratios and Proportional Relationships • Analyze proportional relationships and use them to solve real-world and mathematical problems.
The Number System • Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
Expressions and Equations • Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Key Areas of Focus for Grade 7: Ratios and proportional reasoning; arithmetic of rational numbers

Rationale for Module Sequence in Grade 7:
In Module 1, students build on their Grade 6 experiences with ratios, unit rates, and fraction division to analyze proportional relationships. They decide whether two quantities are in a proportional relationship, identify constants of proportionality, and represent the relationship by equations. These skills are then applied to real-world problems including scale drawings. Students continue to build an understanding of the number line in Module 2 from their work in Grade 6. They learn to add, subtract, multiply, and divide rational numbers. Module 2 includes rational numbers as they appear in expressions and equations—work that is continued in Module 3.

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Sequence of Grade 7 Modules Aligned with the Standards – cont.

Module 3 consolidates and expands students' previous work with generating equivalent expressions and solving equations. Students solve real-life and mathematical problems using numerical and algebraic expressions and equations. Their work with expressions and equations is applied to finding unknown angles and problems involving area, volume, and surface area.

Module 4 parallels Module 1's coverage of ratio and proportion, but this time with a concentration on percent. Problems in this module include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. Additionally, this module includes percent problems about populations, which prepare students for probability models about populations covered in the next module.

In Module 5, students learn to draw inferences about populations based on random samples. Through the study of chance processes, students learn to develop, use and evaluate probability models.

The year concludes with students drawing and constructing geometrical figures in Module 6. They also revisit unknown angle, area, volume, and surface area problems, which now include problems involving percentages of areas or volumes.

Sequence of Grade 8 Modules Aligned with the Standards

- Module 1: The Number System and Properties of Exponents
- Module 2: Congruence
- Module 3: Similarity
- Module 4: Linear Equations
- Module 5: Examples of Functions from Geometry
- Module 6: Linear Functions
- Module 7: Introduction to Irrational Numbers Using Geometry

A breakdown of each Module is available at the following link: http://engageny.org/resource/grades-6-8-mathematics-curriculum-map

Summary of Year:
Eighth grade mathematics is about (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two-and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

CCLS Major Emphasis Clusters
Expressions and Equations • Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations. Functions • Define, evaluate, and compare functions. Geometry • Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem.

Key Areas of Focus for Grade 8: Linear algebra

Rationale for Module Sequence in Grade 8:
This year begins with students extending the properties of exponents to integer exponents in Module 1. They use the number line model to support their understanding of the rational numbers and the number system. The number system is revisited at the end of the year (in Module 7) to develop the real number line through a detailed study of irrational numbers. In Module 2, students study congruence by experimenting with rotations, reflections, and translations of geometrical figures. Their study of congruence culminates with an introduction to the Pythagorean Theorem in which the teacher guides students through the “square-within-a-square” proof of the theorem. Students practice the theorem in real-world applications and mathematical problems throughout the year. (In Module 7, students learn to prove the Pythagorean Theorem on their own and are assessed on that knowledge in that module.)

Sequence of Grade 8 Modules Aligned with the Standards – cont.

The experimental study of rotations, reflections, and translations in Module 2 prepares students for the more complex work of understanding the effects of dilations on geometrical figures in their study of similarity in Module 3. They use similar triangles to solve unknown angle, side length and area problems. Module 3 concludes with revisiting a proof of the Pythagorean Theorem from the perspective of similar triangles.

In Module 4, students use similar triangles learned in Module 3 to explain why the slope of a line is well-defined. Students learn the connection between proportional relationships, lines, and linear equations as they develop ways to represent a line by different equations ($y = mx + b$, $y – y1 = m (x – x1$), etc.). They analyze and solve linear equations and pairs of simultaneous linear equations. The equation of a line provides a natural transition into the idea of a function explored in the next two modules.

Students are introduced to functions in the context of linear equations and area/volume formulas in Module 5. They define, evaluate, and compare functions using equations of lines as a source of linear functions and area and volume formulas as a source of non-linear functions.

In Module 6, students return to linear functions in the context of statistics and probability as bivariate data provides support in the use of linear functions.

By Module 7 students have been using the Pythagorean Theorem for several months. They are sufficiently prepared to learn and explain a proof of the theorem on their own. The Pythagorean Theorem is also used to motivate a discussion of irrational square roots (irrational cube roots are introduced via volume of a sphere). Thus, as the year began with looking at the number system, so it concludes with students understanding irrational numbers and ways to represent them (radicals, non-repeating decimal expansions) on the real number line.

Math Curriculum Structured into 3 Levels of Hierarchy

Curriculum materials for mathematics are structured into 3 levels of hierarchy. The mathematics curriculum structure consists of individual grade levels, with modules divided into lessons.

On the following pages is a common core exemplar for a grade 7 and a high school module. The grade 7 module is for *Addition and Subtraction of Rational Numbers* and the High School module is for *Arithmetic Operations and Polynomials.*

The New York State Common Core Mathematics Curriculum can be accessed at the following link:

http://engageny.org/mathematics

COMMON CORE UNIT:

Straw Man Outline for an Intense Engagement Exemplar in Grade 7

UNIT SUMMARY

Content area: **Addition and subtraction of rational numbers**

Associated CCSS content standards: **7.NS.1, 7.NS.3**

Total instructional time: **15 days**

The rational numbers are an arithmetic system that includes 0 as well as positive and negative whole numbers and fractions. Wherever the term "rational numbers" is used, numbers of all types are implied, including fractions in decimal notation.

The straw man outline presented here does not develop the arithmetic of integers before the arithmetic of rational numbers in general. Rather, each specific phase of the outline would likely begin with integers for simplicity and incorporate other rational numbers second. As with any feature of the straw man, bidders may critique this choice and present alternatives if desired.

Prior knowledge assumed. In grade 6, students learned about signed numbers and what kinds of quantities they can be used to represent. They located them on a number line. As a result of this study, students should have come away thinking of the negative side of the number line as being the mirror reflection of the positive side. For example, by reasoning that the reflection of a reflection is the thing itself, they will have learned that $-(-a) = a$. (Here a may be positive, negative, or zero.) Grade 6 students also learned about absolute value and ordering of rational numbers, including in real-world contexts.

In Grade 6, students will have supported their reasoning about rational numbers with models of rational numbers. These models should continue to function in Grade 7 to support the development of rational arithmetic.

As an aside, the grade 6-to-grade 7 progression for rational numbers is structurally similar to the grade 3-to-grade 4 progression for fractions. In grade 3, students first learned about fractions and what kinds of quantities they can be used to represent. They located them on a number line and ordered them. Then, in grade 4, they began doing arithmetic with these new numbers. Likewise, in grade 7, now that we have some new numbers, we again have to learn how to add, subtract, multiply, and divide with them.

Use of models. Colored chips are commonly used; a similar approach uses small plastic plus signs and minus signs in place of colored chips. Such models have the disadvantage that they do not seem to lend themselves well to fraction reasoning, or consequently to multiplication and division of rational numbers. (Perhaps one could represent a rational number such as $-3/5$ by three chips where one chip represents $-1/5$. In a problem with mixed denominators this would become unwieldy.) If chips are used, it may be especially important for contextual illustrations to involve fractional quantities.

A mathematically robust model for a rational number is an arrow on a number line. To represent a number q, an arrow is drawn from 0 to the point q. The length of the arrow represents the absolute value of the number, and the direction of the arrow indicates the sign of the number. Addition combines arrows tip-to-tail, while multiplication stretches or shrinks arrows by the multiplicative factor, and possibly reverses their direction. This fully represents the scaling/resizing nature of multiplication; see 5.NF.5. (Multiplication with chips seems to revert to an equal-groups picture of multiplication more appropriate to Grade 3; see 3.OA.1.) Other models may be possible.

Whatever models are used, students should represent sums and differences of rational numbers on the number line at various points of the engagement. They should come to recognize, or it can be pointed out to them, that $p + q$ is the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. They should observe, or it should be pointed out to them, that this agrees with addition on the number line as it may have been practiced in earlier grades.

Deductive reasoning, concrete models, and contextual situations. In building up the arithmetic of rational numbers in Grade 7, there is no avoiding a certain amount of deductive reasoning. Students need not deduce everything from definitions and stated assumptions, but neither on the other hand can students reach all of the necessary conclusions by making analogies to everyday situations. That said, everyday situations should certainly be woven into the treatment in order to provide plausibility to results, keep the material interesting, and ensure that students can apply what they learn. In the outline below, a number of calculations are laid out in deductive steps. These calculations are meant to show how the key mathematical ideas fit together. These calculations are not meant to suggest that teachers should fill up the blackboard with equations in the style of a college mathematics course. In each case, one expects that the reasoning will be appropriately motivated and supported by models and contextual situations.

Sequence of ideas and activities

1. Day 1. We review the key grade 6 takeaways, assigning homework with review problems.

2. We work hard on addition for 5 days.

    a. Day 2. We begin with some simple and plausible principles on which others will be built.

        Students are told that addition with signed numbers obeys the same general rules as addition of whole numbers and fractions, including commutativity and associativity. And indeed, **addition of positive numbers agrees with the addition they have been doing all along**.

            Examples:

            $(+3) + (+5) = +8$ agrees with $3 + 5 = 8$.

            $(+1/2) + (+1/3) = +5/6$ agrees with $1/2 + 1/3 = 5/6$.

            Notes. This could be approached in any number of ways - as a logical consequence of a formal definition of addition of rational numbers (along the lines of 7.NS.1.b), as an implication of a model for addition of signed numbers (such as chips or arrows), and/or simply as a useful

thing to say in light of various real-world situations that we model with positive and negative numbers. (For example, if a balloon is 100 meters above sea level and it rises 30 meters, then it is 130 meters above sea level.)

Students are then told that another similarity to what they have done before is that **adding 0 to any number leaves the number unchanged**:

Examples:

$(+1/3) + 0 = +1/3$

$0 + (-8.1) = -8.1$.

Notes. Again this can be approached in any number of ways - as a logical consequence of a formal definition of addition of rational numbers along the lines of 7.NS.1.b, as simple common sense left mostly unremarked upon, as a principle we would like to hang onto going forward into algebra, and/or simply as a useful thing to say in light of various real-world situations that we model with positive and negative numbers. (For example, if someone writes you a check for $0, you have the same amount of credit or debt you had before.)

Classwork and homework: Students solve a variety of word problems in a context involving positive and negative numbers, where the problem calls for addition of two positive numbers or addition of zero to a number.

b. Day 3. Students then work on problems like:

$(+10.3) + (-10.3) = 0$

and

$(-2/3) + -(-2/3)$

$= (-2/3) + (+2/3)$

$= 0$

to learn that **a number and its opposite sum to zero**. Again this can be approached in any number of ways - as a logical consequence of a formal definition of addition of rational numbers along the lines of 7.NS.1.b, as an implication of a model for addition of signed numbers such as chips or arrows, and/or simply as a useful thing to say in light of various real-world situations that we model with positive and negative numbers. (For example, if you win $10.30 and then lose $10.30, you haven't gained or lost anything. Another example, which would require appropriate scientific knowledge, is that a positive proton and a negative electron combine to form a neutral atom.)

Classwork and homework: Students work on some pure computational problems, and some word problems, in which a number and its opposite are added. These might include problems that call for the addition of three numbers, two of which are opposites of one another.

c. Day 4. Students find the **sum of negative numbers** using a combination of common sense, models and/or definitions, and what has been learned.

To find: (–7) + (–4) = ?

Students could use a model of addition to find the sum –11, and also think of the problem contextually: If it is already 7 degrees below zero, and the temperature drops another 4 degrees, how cold is it now? 11 degrees below zero.

Classwork and homework: After doing a few of these, students will catch on to the general pattern for adding two negative numbers. They practice computation problems and word problems that include fractions and decimals, and perhaps problems with three or more negative addends. Practice should also mix in some problems of the types previously encountered.

d. Day 5. Students find the **sum of a positive number and a negative number**. This is the first really difficult moment in the sequence. Again this is a combination of common sense, models and/or definitions, and what has been learned.

Examples

(+700) + (–100)

= (+600) + (+100) + (–100) because (+600) + (+100) = +700

= (+600) + 0 by property of adding opposites

= +600 by property of zero.

(+50) + (–80) = (+50) + (–50) + (–30) = 0 + (–30) = –30.

(This "neutral pair" approach is meant to show where the answer comes from; it is not necessarily an efficient algorithm for adding rational numbers.)

Students are told the **general pattern for the results of addition**, if they haven't figured it out. When adding two numbers with different signs, the number with smaller absolute value partially cancels the number with larger absolute value. The sum has the same sign as the addend with larger absolute value, but the sum is closer to 0 than that addend is.

e. Day 6: **Practice**. No new ideas are introduced. Students do computational and word problems that call for adding three or more signed numbers of various signs and relative absolute values, including fractions and decimals. (Students may use number line representations to support reasoning if they like, but the problems themselves do not refer to representations or models.)The two goals are fluency in computation and confidence and correctness in application.

Toward the end of this work, we stop using the + symbol to indicate positive numbers.

3. We now turn to subtraction. Another 4 days of hard thinking.

a. Day 7. As was the case for addition, we begin with simple and plausible principles on which others are built.

Students are reminded that in their previous study of subtraction, they check their answer to subtraction by adding. In general, we check our answer to $a - b = c$ by adding $b + c$ and making sure we get back to a. This is still true for rational numbers. If ever in doubt about a subtraction, we check our results by adding.

Students are told some important special cases of this: first, **subtracting anything from itself gives zero**: $a - a = 0$. This would probably just strike most students as common sense. It is true because if you add $a + 0$ you get back to a.

Also, **subtracting zero leaves a number unchanged**: $x - 0 = x$. Again plausible, and true because if you add $x + 0$ you get back to x. The teacher does a few numerical examples at the board to make sure people are good with this. The presentation includes contextual examples in addition to the above mathematical reasoning.

These special cases are so plausible that some students might like them perfectly well without completely following the subtle argument about adding back; such students are not browbeaten with the reasoning but are simply allowed to embrace the results willingly.

b. Day 8. Students learn the general rule for subtraction, $a - b = a + (-b)$. **Subtracting means adding the opposite**.

During their work with addition, students likely already noticed special cases of the general rule. For example, in (+10) + (–3), the answer +7 was like subtracting 3 from 10. Students collect these suggestive examples and observe that subtractions familiar from previous grades can be recast as adding the opposite. For example, $4/5 - 1/5 = 3/5$ is familiar from previous grades, and more recently one may recall the result of $4/5 + (-1/5)$ as being $3/5$ as well. So in this case, $4/5 - 1/5 = 4/5 + (-1/5)$. From such examples, supported as appropriate by models and contextual situations, students conjecture, or are led to conjecture, that all subtraction of rational numbers is adding the opposite.

Students make a small test of this conjecture by showing how this rule agrees what came immediately before: namely, it reproduces the prior conclusions that $a - a = 0$ (because $a + (-a) = 0$) and that $x - 0 = x$ (because $x + (-0) = x + 0 = x$).

Examples like the following are then used to motivate/derive/justify the general rule. Each example given might be motivated by a context, then modeled, then formalized; or treated like a logical argument and then modeled and illustrated in context, to "see it in action."

Example:

$3 - 5$

$= 3 + 0 - 5$ by the property of 0

$= 3 + (-5) + 5 - 5$ by the property of adding opposites

$= 3 + (-5) +$ "$a - a$" noticing an instance of one of the special cases above

$= 3 + (-5) + 0$ by the above special case

$= 3 + (-5)$ by the property of 0; this step shows the general rule

$= -2.$ by previous work.

As with any subtraction problem, we check the answer by addition:

$5 + (-2)$

$= 3.$ It checks. by previous work on adding rational numbers.

Another Example

$-8 - (-12)$

$= -8 + 0 - (-12)$ by the property of 0

$= -8 + 12 + (-12) - (-12)$ by the property of adding opposites

$= -8 + 12 +$ "$a - a$" noticing an instance of one of the special cases

$= -8 + 12 + 0$ by the special case

$= -8 + 12$ by the property of 0; this step shows the general rule

$= 4.$ by previous work.

As with any subtraction problem, we check the answer by addition:

$-12 + 4$

$= -8$. It checks. by previous work on adding rational numbers.

Aside: Here is the first example another way:

$3 - 5$

$= 3 + 0 - 5$ by the property of 0

$= 3 + 2 + (-2) - 5$ by the property of adding opposites

$= 5 + (-2) - 5$ by previous work

$= 5 - 5 + (-2)$ by properties of addition

$=$ "$a - a$" $+ (-2)$ noticing an instance of one of the special cases above

$= 0 + (-2)$ by the above special case

$= -2$. by previous work.

Compared to the first calculation shown for $3 - 5$, this is probably an easier way to get to the answer, because it takes students there by a reassuringly familiar path of "taking small from big." However, the point of having this discussion was not to train students in an optimal method for getting the answer to problems like $3 - 5$. The point was to motivate, illustrate, or justify the fact that subtraction is adding the opposite. Observe that in the second method for $3 - 5$, this general rule does not appear.

Classwork and homework: Students practice subtraction of two rational numbers, including fractions and decimals. In some cases they are asked to check the results by addition.

c. Day 9: Students work with **parentheses**. The first problem they do is –(8+5) = ?. This is easy if we begin inside the parentheses, 8+5 = 13 so –(8+5) = –13. Students are shown this result in a new light: because –13 = –8 + –5, our result can be written as –(8+5) = –8 + –5. This can be described by saying that **"the opposite of a sum is the sum of the opposites."** This can be illustrated with models and contextual situations. Students verify this for all of the possibilities with signs. The principle is then extended to the case of three or more summands. Students might be invited to think of this as "distributing in the minus sign." (The use of the term "distributing" will be justified later during the study of multiplication of rational numbers, when we find that –1 times a number is the opposite of the number: $(-1)(a) = -a$. Then the principle that the opposite of a sum is the sum of the opposites can be seen from the distributive property as $-(b + c) = (-1)(b + c) = (-1)b + (-1)c = -b + -c$.)

Next problem: 6 – (8+5) = ? We can do this as 6 – (8+5) = 6 – 13 = 6 + (–13) = –7. In that approach, we evaluate parentheses first as usual and then apply the principle that subtraction is adding the opposite. Or, we can view the term in parentheses as a "chunk" and apply the subtraction principle first: 6 – (8+5) = 6 + –(8+5). As a second step, we can evaluate the term in parentheses; this leads to 6 + (–13) as before. Alternatively, as a second step we can handle the term in parentheses by applying the principle that the opposite of the sum is the sum of the opposites:

$$6 + \underline{-(8+5)} = 6 + \underline{(-8) + (-5)}\ .$$

Now we have a sum of three summands, 6 + (–8) + (–5). This can be evaluated in various ways, e.g., adding the first two terms first gives (–2) + (–5) = –7.

Classwork and homework: Students solidify this by evaluating simple numerical expressions with sums and differences of positive and negative rational numbers that involve parentheses.

d. Day 10: **Practice**. Students do computational problems involving addition and subtraction of three or more rational numbers, including positive and negative fractions and decimals, problems with parentheses, and problems with more than one subtraction operation. The goal is fluency.

4. We now turn to finding distances on the number line.

Students in Grade 6 learned that the absolute value of a number is its distance from 0 on a number line. They also found the distance between two points in the coordinate plane having the same first coordinate or the same second coordinate; they will have done this graphically, because in grade 6 they did not have a general principle for finding such distances. In grade 7, they will use subtraction in such a general principle.

a. Day 11. In contextual settings, students find the distance between two numbers on the number line graphically, in integer cases as well as in cases such as $-4\ 1/3$ and $-2/3$ that may induce them to connect the problem of finding distance to the operation of subtraction.

Students realize, or it is pointed out to them, that **the distance is found by taking the absolute value of the difference between the two numbers in either order, $|a - b|$, or, by subtracting the lesser number from the greater**. This can then be justified by appeal to models or definitions.

Students show, or are shown, that this principle agrees with distances found on the number line in earlier grades. Students also realize, or it is pointed out to them, that this principle is consistent with what they learned in Grade 6: at that time they learned, as a matter of definition, that the distance between 0 and $a$is$|a|$. The new principle is consistent, because it says that the distance between 0 and a is given by subtraction as $|a - 0| = |a|$.

b. Day 12: No new mathematics is introduced. Students apply the subtraction principle to solve word problems involving the distance between two numbers, such as, a weather balloon is 100,000 feet above sea level, and a submarine is 3 miles below sea level. How high above the submarine is the weather balloon? John was $3.75 in debt, and Mary was $0.50 ahead. John found an envelope with some money in it, and after that he had the

same amount of money as Mary. How much was in the envelope? On their way to giving the answer, students should represent these problems by a math diagram, a number line, and an equation.

c. Day 13: Students apply the distance principle in word problems to find distances in the coordinate plane computationally for points with the same first coordinate or second coordinate, as they did graphically in grade 6.

d. Day 14: **Mixed practice**. Students practice problems along the lines of 2(e), 3(d), 4(b), 4(c).

5. Day 15: Unit test combining 2(e), 3(d), 4(b), 4(c). A score of 90% is required to pass.

Further developments during Grade 7

From this point on, algebra work with expressions and equations should sometimes involve positive and negative rational numbers, including positive and negative fractions and decimals, as well as parentheses. Also, applied percentage problems should sometimes involve negative percent differences.

COMMON CORE UNIT

Straw man outline for an intense engagement in High School Algebra

Content area: **Arithmetic Operations on Polynomials**

Associated CCSS content standards: **A-APR.1**

Total instructional time: **7 days**

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. Viewing an expression as the result of operations on simpler expressions can sometimes clarify its underlying structure.

This high school module is intended to extend and connect students' understandings from middle school to polynomials. It focuses on reinforcing knowledge and skills, rather than developing completely new knowledge and skills.

Prior knowledge assumed. Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions. By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving. In high school algebra, these aspects carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."

In Grade 5, students will have written and interpreted numerical expressions, gaining familiarity with such mathematical notations as parentheses and brackets. In grade 6, they are expected to apply and extend their previous understanding of arithmetic to algebraic expressions by writing, reading, and evaluating expressions in which letters stand for numbers. They are expected to apply the properties of operations to numerical and simple algebraic expressions to come up with equivalent expressions. By grade 7, the ability to use properties of operations to generate equivalent expressions is extended specifically to adding, subtracting, factoring, and expanding linear expressions with rational coefficients. Students in grade 7 are also expected to be able to rewrite an expression in different forms to shed light on the problem and how the quantities in it are related.

The Standards for Mathematical Practice are important aspects of teaching and learning mathematics across all of the grades. As students in middle school use properties of operations to rewrite expressions and generate equivalent expressions that are helpful to them in solving problems, they need to look for and make use of structure. For example, as they progress through the grades, they can see increasingly complicated things that they need to make sense of.

Use of instructional tools.

Algebra tiles can be useful for visual representation of algebraic expressions and operations to combine them.

Technologies, such as student response systems, interactive white boards, blogging sites, and interactive websites, can be useful teaching and learning tools.

Deductive reasoning, concrete models, and contextual situations. In building up arithmetic operations on polynomials in high school, there is no avoiding a certain amount of deductive reasoning. Students need not deduce everything from definitions and stated assumptions, but neither on the other hand can students reach all of the necessary conclusions by making analogies to everyday situations. That said, everyday situations should certainly be woven into the treatment in order to provide plausibility to results, keep the material interesting, and ensure that students can apply what they learn. In the sequence of ideas and activities below, a number of calculations are laid out in deductive steps. These calculations are meant to show the reader how the key mathematical ideas are intended to fit together. These calculations are not meant to suggest that teachers should fill up the blackboard with equations in the style of a college mathematics course. One expects that the learning as conducted in class will be appropriately motivated and supported by models and contextual situations.

Standards for Mathematical Practice. All eight Standards for Mathematical Practice will be used during this unit. However, there are certain Standards for Mathematical Practice that are particularly applicable to the sequence of activities described for this unit.

- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Sequence of ideas and activities

1. Day 1. We review number systems and their properties that students worked with in middle school, stressing the coherence of these properties across number systems and their applicability to increasingly sophisticated systems. Have students use properties to justify the equivalences of numeric and simple algebraic expressions. Terms to reviewed/discussed on Day 1 include monomial, term, variable, and the various properties (i.e., commutative, associative, distributive, identity, etc.). Assign homework problems that bridge students to the concept of polynomials.

   ***Examples*:**

   Review activities for properties of real numbers might include problems like the following:

   The statement $3x + 3y = 3(x+y)$ uses which property of real numbers?

   The statement $(a + b) + c = a + (b+c)$ illustrates which property of real numbers?

Review activities for operations with rational numbers might include problems like the following. As students begin work with operations on polynomials, it is important that they have a solid grasp on performing operations with rational numbers (in particular, addition, subtraction, and multiplication).

Perform the indicated operation:

-25 + 32

-18 – (-15)

(-5)(8)(-10)

-8[-5+(-9)]

Sample problems to be worked in class as a whole group that involve determining equivalent numeric and algebraic expressions and the identification of the properties and rules that justify the equivalences.

Identify the mathematical property or rule that justifies each step below.

5[2 + (3+1) – 6(4 + 7) + 8]

5[(2 +3) + 1 – 6(4 + 7) + 8]

5[(2 + 3) +1 – 24 – 42 + 8]

5[5 + 1 – 24 -42 +8]

5[-52]

-260

3[2(a + b) + 4a]

3[4a + 2(a + b)]

3[4a + 2a + 2b]

3[6a + 2b]

18a + 6b

Classwork and homework: In addition to practice problems, students could be asked to create their own multi-step algebraic or numeric expression and a set of defined steps to arrive at an equivalent expression. Then on Day 2, students could do a "think, pair, share" activity where they exchange expressions and identify the properties or rules used for each step. In addition, students would be expected to check their partner's precision (Mathematical Practice #6).

2. Day 2.

   a. We work with students to build understanding of what a polynomial is—building from the notion of monomial. Make students aware of special cases of polynomials like binomials and trinomials.

   b. Students are introduced to addition of polynomials, extending their understanding of properties of addition, including the identity element.

   Notes: *Teacher will begin class going over homework practice problems and engaging students in the "think, pair, share" activity. This will take about 15 minutes. Teacher will then define terms for specific types of polynomials (e.g., monomial, binomial, and trinomial). This will then lead to a discussion of polynomials and the more general definition that characterizes polynomials with more than 3 terms (even though monomials, binomials, and trinomials are, of course polynomials).*

   Examples:

   Generally, students will complete problems like the following.

   (5y – 7) + (-1 + 4y)

   $(-x^3 + 7x^2 - 2) + (2x^3 + 2x^2 - 5x)$

   Classwork and homework: Students will complete problems like the ones above as a class and then individually. Students will share their work with the class and explain how they arrived at the answer. Homework will include problems similar to the classwork but also including a problem like the one below:

   $3[2(4x^2 - 5)] + 2[2(x^2 + 1)]$

3. Day 3. Students are introduced to subtraction of polynomials, extending their understanding of properties of subtraction, including the concept of the additive inverse.

   Notes: Teacher can refer back to the material covered on the first day to review the rules of subtraction with rational numbers as a segue into working with polynomials. Teachers will explain to the students that any subtraction problem has an equivalent addition problem. Teachers can also show the students that these problems can be done horizontally as well as vertically.

Examples:

Generally, students will complete problems like the following.

(6a + 2b - 3c) – (4a - b - 2c)

x – (2x – 3)

It is also important for students to be able to set up the problem by themselves like the example below:

If $x^2 - 3y + 5$ is subtracted from $2x^2 - 4y + 3$ the result is:

Classwork and homework: Students will complete problems like the ones above as a class and then individually. Students will share their work with the class and explain how they arrived at the answer. Homework will include problems similar to the classwork but also include a written assignment asking the students to explain how a subtraction problem became an addition problem and why is this mathematically correct. Students can research this topic but make sure to include citations where appropriate.

4. Day 4.

   a. Review properties of exponents in relation to monomials

   b. Develop the conceptual understanding of multiplication of polynomials, by extending students' understanding of properties of multiplication, with emphasis on application of the distributive property. Focus will be on multiplication of a polynomial by a monomial and an introduction to multiplication of binomials.

Notes: Teacher will review the properties of exponents at the beginning of class this can be done by showing $x^3 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x = x^5$. These problems could include a coefficient of a decimal or fraction.

Examples: To scaffold this lesson the following types of questions will be covered in the order below:

$3x \cdot x$

$-2xy^3(4x^2y^3)$

$2a^2(a^2 + 3a + 5)$

$(y^3 + 4y - 5)y$

In the accompanying diagram, the width of the rectangle is represented by $2x^2$ and the length is represented by 3x+1.

3x+1

a) Express the area of the rectangle in terms of x.

b) Express the perimeter of the rectangle in terms of x.

Classwork and homework: Students will complete problems like the ones above as a class and then in small groups. Students will share their work amongst their group and then will report out with the class and explain how they arrived at the answer. Homework will include problems similar to the classwork but also including a problem like the ones below:

$(-3x)(2x)(-4x^3)$

Students will also be asked to solve the following problem: (x+2)(2x-3) Students will be asked to come up with an solution as well as a justification of the solution. Then on Day 5, students could do a "think, pair, share" activity where they compare their work. If teachers create a blog, they can post the multiplication problem on the blog and then students could collaborate and share their ideas to work together to come up with a solution. Students should not be discouraged to research this topic on the internet or in a textbook.

5. Day 5. Continue to develop students' conceptual understanding of multiplication of polynomials. Again, emphasize application of the distributive property. Focus will be on multiplication of two binomials and the multiplication of polynomials.

Notes: Teacher will begin class going over the homework problems and engaging students in the "think, pair, share" activity. Teacher will ask for students to share their solutions and explanation to their work. This will encourage students to converse about their solutions. This will take about 15 minutes.

Examples:

Generally, students will complete problems like the following:

$(3x + 2)(x - 1)$

$(2x^3 - 3)(4x + 5)$

Students will also complete problems like the following:

(y+5)(y-5)

$(3x - 1)(3x + 1)$

Students will be asked what they notice about the product of these two binomials versus the previous ones completed. Students should discover this rule.

Classwork and homework: Students will complete problems like the ones above as a class and then in small groups. Students should think individually about the special pair of binomials and then break into groups and discuss their thoughts and then will report out with the class. Homework will include problems similar to the classwork but also including a problem like the ones below:

Express the area of the square in terms of x.  (2x + 2)

6. Day 6. The focus will be application-related work or review that integrates the concepts of addition, subtraction, and multiplication of polynomials. These activities will provide students with reinforcement and also the opportunity to be engaged in mixed practice. No new mathematics is introduced. Students will work to solve multi-stop word problems involving geometric figures.

   Classwork: This day could be spent having students play a pre-planned review game or break students into groups and assign each group an operation and have the students teach a 5 minute mini review of what has been covered throughout this unit. On this day the teacher could also use a student response system to find out the students level of understanding.

6. Day 7: Unit test including basic computational skills with polynomials along with problems that will require students to demonstrate their conceptual understanding and the ability to translate to an applied problem.

<u>Further developments during High School</u>

From this point on, arithmetic operations with polynomials can and should be imbedded in future algebra work with expressions and equations. For example, as students work to solve equations and systems of equations, they should be expected to use the arithmetic operations of addition, subtraction, and multiplication to find equivalent expressions. In addition, students should be able to examine a real-world problem and decontextualize it into an algebraic expression or equation. From that, they will be able to use the properties of operations with polynomials to determine the solution.

SAMPLE PLANS 6

Mathematics 6-12

ALIGNED TO COMMON CORE

Middle School Mathematics

Providing high quality, highly effective instruction and equal access to academic opportunities is the most important service we can provide our students. Ensuring that all students have comparable academic programming options has been a primary driver for many of the scheduling and programmatic changes from the 2012/13 to the 2013/14 school year. As the district continues to build an infrastructure that better supports all schools, we will realize the promise of offering engaging instructional opportunities via rigorous curriculum and content and consistent academic programming for every child, in every classroom, every day.

Core Mathematics Curriculum

Teachers in Grade 6 - 8 will use the APW developed units in conjunction with district-adopted curricular resources until the NYS Core Curriculum is implemented. The APW scope and sequence for mathematics accounts for key grade-level focus areas determined by the Common Core Learning Standards. We aimed to significantly narrow and deepen the scope and content of how time and energy is spent in the math classroom. This increased focus allows each student to think, practice, and integrate each new idea into a growing structure. Each unit provides teachers with guidance about lessons, projects, or tasks to help them plan daily instruction. As teachers plan they should account for the following lesson components and instructional shifts.

Fluency:

The Common Core Learning Standards explicitly call for fast and accurate computation. Fluency is best addressed through short daily routines such as timed fact test, mental math exercises and Number Talks. Students in the middle grades should spend approximately 10 minutes a day practicing the following crucial fluencies. The table to the right is the end of year fluency expectations required by the Common Core Learning Standards. Teachers are encouraged to identify additional fluencies that will aid in mastery of the standards.

Grade	Required Fluency
6	Multi-digit division Multi-digit decimal operations
7	Solve $px + q = r$, $p(x + q) = r$
8	Solve simple 2x2 systems by inspection

Conceptual Understanding:

Teachers must support the development of deep conceptual understanding, not just algorithms and answer-getting, in their students. Class time should be structured to support students' ability to access concepts from a number of perspectives. Teachers are encouraged to use a variety of instructional strategies to build mathematical knowledge so students see math as more than a set of discrete procedures. Teacher should spend the majority of class time – approximately 40 minutes – developing conceptual understanding.

Application:

Students demonstrate deep conceptual understanding of core math concepts by applying them to new situations. Teachers must devote time for students to use math and choose the appropriate procedure for application without prompting. Mathematics classes should establish a connection between math and the real world. Solving problems in context is what will build strong mathematicians. Students should spend 10-15 minutes a day applying the math they've learned.

Grade 6 Sample Lesson

Estimated Time: 60 minutes

Domain: Ratio & Proportional Relationships

Cluster: Understand ratio concepts and use ratio reasoning to solve problems.

CCS Standard: 6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Mathematical Practice(s): 2. Reason abstractly and quantitatively.

Key Vocabulary: Unit Rate, Rate Reasoning, Unit Pricing, Constant Speed, Ratio

Essential Question: How do we use ratios to solve real-world unit rate problems?

The Bridge:
Remind the students of the applications of ratios to real world problems. Ratios are used to compare two things using numbers.
Pose the following question to the class:

"*Jackie bought three equally-priced dresses at the store. She paid $150.00 for all three dresses. How much did she pay per dress?"*

Ask students for their thoughts on how to solve this problem. (Possible strategies could include: division, charts, tree diagrams, etc.)

The Mini Lesson:
To solve this you must know how much was paid in total and how many items were purchased. In this case she purchased 3 dresses (the quantity) for $150.00 (the total amount).

*****the most direct strategy to solve this problem would be division*****

Students should complete the division to solve the problem. **Jackie paid $50 per dress.**

Now that students know the correct answer model for them how to solve this problem using ratio reasoning…

$\frac{3}{150}$ **The number of dresses to the total amount spent; the unit rate is** $\frac{1}{50}$ **one dress for $50**

Pose the following question "How many dresses could be bought for $250?"

Have students work in pairs to solve this. Guide students to create a ratio.

Work Period:
Split the students into 4 groups and assign each group a portion of the Running at Constant Speed problem attached at the bottom of this page (1-4)
After 15 minutes have groups create a chart that explains and outlines the steps used to solve their portion of the problem, similar to reciprocal teaching. Each group will then report out and present their findings

Summary:
After each group has presented, select a few students to explain how they set up a ratio to solve the problem as well as any conversions they may have needed to make (ie: hours to minutes). Post the charts the groups have created on the wall.

Closure:
Assign the following Homework Problems

1. Over a period of 3 hours, 180 leaves fell from a tree. At this rate, how many leaves fell in one hour?
2. Georgia drove a total of 252 miles and used 12 gallons of gasoline. What is this rate in miles per gallon?
3. Tyler scored 21 goals in 7 soccer games. At this rate, about how many goals did he score each game?
4. While climbing down a mountain, Anthony descended 45 feet every hour. At this rate, how many feet will he descend in 6 hours?

Student sheet is attached at the bottom of this lesson

Learning Extensions:

Additional Resources: **Task with student work samples**

Use the problem attached at the end of the lesson for an additional resource.

6.1.3.7 Work period

Name _____ Date_____

Task: 6.1.3.7 Running at Constant Speed Work Period CCSSM 6.RP.3b

A runner ran 20 miles in 150 minutes. If she runs at that speed,

1. How long would it take her to run 6 miles?
2. How far could she run in 15 minutes?
3. How fast is she running in miles per hour?
4. What is her pace in minutes per mile?

Alignment 1:6.RP.3

Grade

6

Domain

RP: Ratios and Proportional Relationships

Cluster

Understand ratio concepts and use ratio reasoning to solve problems.

Standard

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Commentary: None

Solution:

Solution: Using a table

	A	B	C	D	E	F
Number of Minutes	150	15	7.5	30	45	60
Number of Miles	20	2	1	4	6	8

The values in column B were found by dividing both values in column A by 10. The values in column C were found by dividing both values in column B by 2. The other columns contain multiples of the values in column B.

a. If we look in column E, we can see that it would take her 45 minutes to run 6 miles.

b. If we look in column B, we can see that she could run 2 miles in 15 minutes.

c. If we look in column F, we can see that she is running 8 miles every 60 minutes (which is 1 hour), so she is running 8 miles per hour.

d. If we look in column C, we can see that her pace is 7.5 minutes per mile.

6.1.3.7 Homework

Name _____ Date_____

6.1.3.7 Homework

1. Over a period of 3 hours, 180 leaves fell from a tree. At this rate, how many leaves fell in one hour?

2. Georgia drove a total of 252 miles and used 12 gallons of gasoline. What is this rate in miles per gallon?

3. Tyler scored 21 goals in 7 soccer games. At this rate, about how many goals did he score each game?

4. While climbing down a mountain, Anthony descended 45 feet every hour. At this rate, how many feet will he descend in 6 hours?

6.1.3.7 Additional Resource

Name _____ Date_____

Task: 6.1.3.7 Price per Pound and Pounds Per Dollar CCSSM 6.RP.3

The grocery store sells beans in bulk. The grocer's sign above the beans says,

5 pounds for $4.

At this store, you can buy any number of pounds of beans at this same rate, and all prices include tax.

Alberto said,

> *“The ratio of the number of dollars to the number of pounds is 4:5. That's $0.80 per pound.”*

Beth said,

> *"The sign says the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar."*

a. Are Alberto and Beth both correct? Explain.
b. Claude needs two pounds of beans to make soup. Show Claude how much money he will need.
c. Dora has $10 and wants to stock up on beans. Show Dora how many pounds of beans she can buy.
d. Do you prefer to answer parts (b) and (c) using Alberto's rate of $0.80 per pound, using Beth's rate of 1.25 pounds per dollar, or using another strategy? Explain.

Alignment 1:6.RP.2

Grade
6
Domain
RP: Ratios and Proportional Relationships
Cluster
Understand ratio concepts and use ratio reasoning to solve problems.
Standard
Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.” Expectations for unit rates in this grade are limited to non-complex fractions.

Commentary:

This task could be used by teachers to help students develop the concept of unit rates. Its purpose is to help students see that when you have a context that can be modeled with a ratio and associated unit rate, there is almost always another ratio with its associated unit rate (the only exception is when one of the quantities is zero), and to encourage students to flexibly choose either unit rate depending on the question at hand.

Item (d) admits many different answers and is intended to prompt a teacher-facilitated discussion of different student strategies. A productive discussion could develop around side-by-side comparisons of strategies that apply Alberto's rate and strategies that apply Beth's rate.

Solution:

Solution: Using a ratio table

(a) Alberto and Beth are both correct. Their rates could be illustrated with a double number line or a ratio table like the following:

Pounds	Dollars
1	.80
1.25	1
2.5	2
5	4

(b) Double the quantities in Alberto's rate to find the price of two pounds:

Pounds	Dollars
1	.80
2	1.60

(c) Starting from Beth's rate and multiplying both quantities by ten shows the number of pounds that can be purchased for 10 dollars:

Pounds	Dollars
1.25	1
12.50	10

(d) Answers may vary. We can efficiently answer part (b) using Alberto's rate and part (c) using Beth's rate.

Grade 7 Sample Lesson

(proportional vs. non proportional)

Time Frame: 90 minutes

Domain: Ratios and Proportional Relationships

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard:
7RP.2.A: Decide whether two quantities are in a proportional relationship.
7RP.2.C: Represent proportional relationships by equations.
7RP.2.D: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

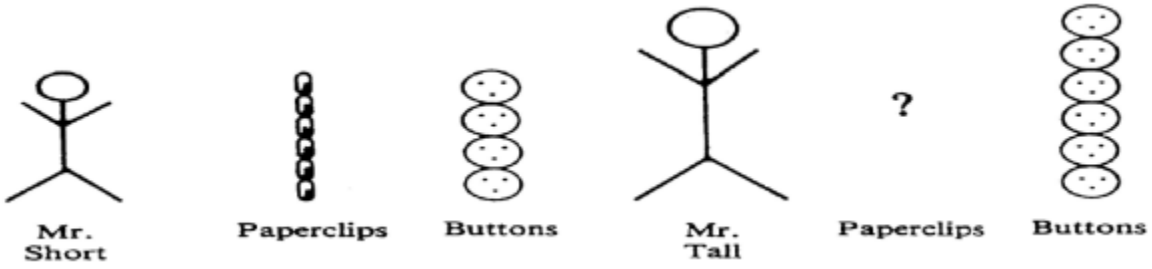
Mathematical Practices:
Model with mathematics; Look for and make use of structure; Look for and express regularity in repeated reasoning

Vocabulary: ratio; proportion; proportional relationship, constant of proportionality = unit rate

Materials: large visual of Bridge picture

Essential Question:
What are the properties of a proportional relationship and what are the properties of a non-proportional relationship?

The Bridge: In the picture, they use paperclips and buttons to measure Mr. Small. Now you use the buttons to measure Mr. Tall. How many paperclips tall is Mr. Tall? How do you know, explain your answer.
(Teachers: Most students will say Mr. Tall is eight paperclips tall. The misconception is: they added two more buttons equivalent to the height of Mr. Tall therefore they must also add two more paperclips to measure Mr. Tall. Actually, there are three paperclips to every two buttons equal to the height of Mr. Short. Therefore, you should add another three paperclips to equal the height of Mr. Tall. This would be a total of nine paperclips to measure Mr. Tall, not eight paperclips.)



(Teacher Notes)

Read the problem together and discuss questions as you examine different representations of the word problem in data tables, in graphs and in equations. Guide the students in exploring the properties of a proportional relationship and the properties of a non-proportional relationship.

Jet Ski Rentals

Adriana has an opportunity to go jet skiing on Canandaigua Lake for the 4th of July. She looked up the prices to rent a jet ski for the long weekend. Below is the information Adriana gathered from two different Jet Ski Rental companies: Canandaigua Jets and the Jackson Jet Ski company. You are going to explore what might be the properties of a proportional relationship and what might be the properties of a non-proportional relationship, Use the data tables below to help you answer some questions.

Work Period:

A. USING TABLES TO DETERMINE PROPORTIONALITY

Calculate the ratio of $\frac{y}{x}, \left(\frac{\text{cost}}{hour}\right)$ in each data table. This is the unit rate. Then answer the questions below.

Canandaigua Jets

(X) (Y)

NUMBER OF HOURS	TOTAL COST ($)	RATIO: $\frac{y}{x}$
1	$45	
2	$90	
3	$135	
4	$180	
5	$225	

Jackson Jet Ski Company

(X) (Y)

NUMBER OF HOURS	TOTAL COST ($)	RATIO: $\frac{y}{x}$
1	$75	
2	$120	
3	$165	
4	$210	
5	$255	

Fill is the equations for this table.

1] How are the tables alike? _____

2] How are they different? _____

3] Which one is proportional? _____

4] What makes it a proportional relationship? _____

CONCLUSION:

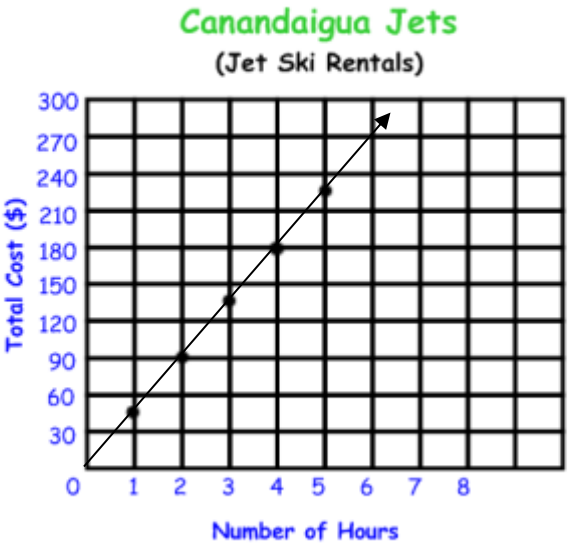
To determine proportionality from a table you _____

_____

_____

B. **USING GRAPHS TO DETERMINE PROPORTIONALITY**

Look at the graphs of Canandaigua Jet Ski Company and Jackson Jet Ski Company. Make observations of each graph and answer the questions below.

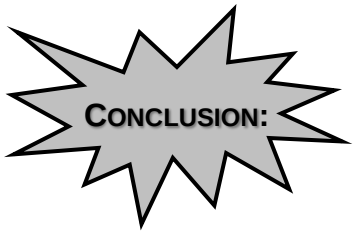


1] How are the graphs alike?

2] How are they different?

3] Which one is proportional?

4] What makes it a proportional relationship?



To determine proportionality from a graph, _____

C. USING EQUATIONS TO DETERMINE PROPORTIONALITY

Canandaigua Jets Company **Jackson Jet Ski Company**

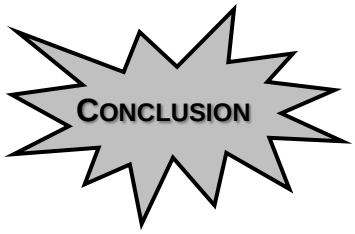
$y = 45x$ $y = 45x + 30$

1] How are the equations alike?

2] How are they different?

3] Which one is proportional?

4] What makes it a proportional relationship?



To determine proportionality from an equation, _____

_____

_____

Summary:
Facilitate a class discussion reviewing what the properties of a proportional relationship are and the properties of a non-proportional relationship in: a table; graph; and in equation.

Closure:
Journal write: students write one example of a proportional relationship and one example of a non-proportional relationship that they are familiar with from their everyday lives. (Examples can be used for homework.)

Name_____ **Date** _____

Hwk 7.2.3.1 Proportional Reasoning

Definition of Proportional Relationship

Facts/ Characteristics

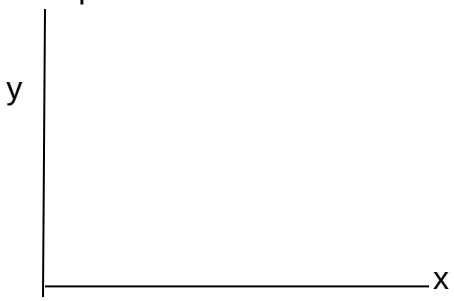
Proportional Relationships

Examples of Proportionality

1) Table

x y Ratio $\frac{y}{x}$

2) Graph



3) Equation _____

Non-Examples of Proportionality

1) Table

x y Ratio $\frac{y}{x}$

2) Graph

3) Equation _____

Name _____
Date_____

7.2.2.2 CWK

Jet Ski Rentals

Adriana has an opportunity to go jet skiing on Canandaigua Lake for the 4th of July. She looked up the prices to rent a jet ski for the long weekend. Below is the information Adriana gathered from two different Jet Ski Rental companies: Canandaigua Jets and the Jackson Jet Ski company. You are going to explore what might be the properties of a proportional relationship and what might be the properties of a non-proportional relationship, Use the data tables below to help you answer some questions.

A. USING TABLES TO DETERMINE PROPORTIONALITY

Calculate the ratio of $\frac{y}{x}, \left(\frac{cost}{hour}\right)$ in each data table. This is the unit rate. Then answer the questions below.

Canandaigua Jets

(X) (Y)

NUMBER OF HOURS	TOTAL COST ($)	RATIO: $\frac{y}{x}$
1	$45	
2	$90	
3	$135	
4	$180	
5	$225	

Jackson Jet Ski Company

(X) (Y)

NUMBER OF HOURS	TOTAL COST ($)	RATIO: $\frac{y}{x}$
1	$75	
2	$120	
3	$165	
4	$210	
5	$255	

Fill in the equations for this table.

1] How are the tables alike? _____

2] How are they different? _____

3] Which one is proportional? _____

4] What makes it a proportional relationship? _____

CONCLUSION:

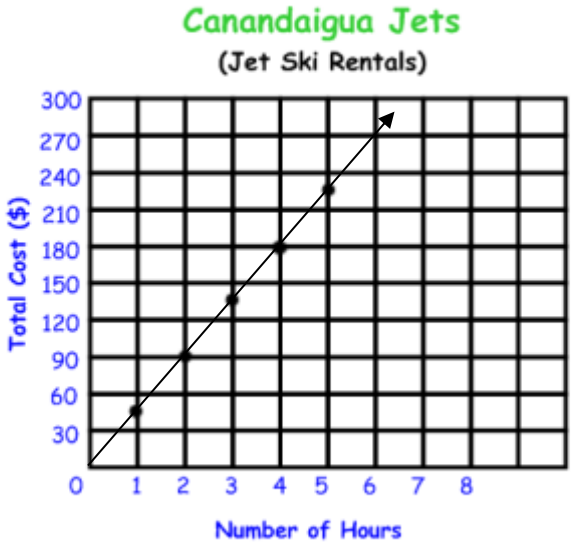
To determine proportionality from a table you _____

_____

_____

B. **USING GRAPHS TO DETERMINE PROPORTIONALITY**

Look at the graphs of Canandaigua Jet Ski Company and Jackson Jet Ski Company. Make observations of each graph and answer the questions below.

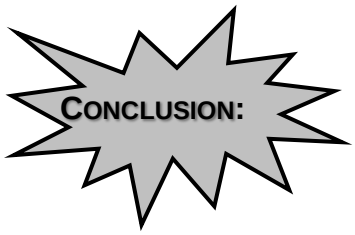


1] How are the graphs alike?

2] How are they different?

3] Which one is proportional?

4] What makes it a proportional relationship?



To determine proportionality from a graph, _____

_____

_____

C. USING EQUATIONS TO DETERMINE PROPORTIONALITY

Canandaigua Jets Company **Jackson Jet Ski Company**

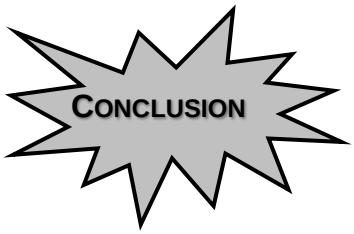
$y = 45x$ $y = 45x + 30$

1] How are the equations alike?

2] How are they different?

3] Which one is proportional?

4] What makes it a proportional relationship?



To determine proportionality from an equation, _____

_____

_____

Name _____
Date_____

7.2.2.2 HWK

Definition of Proportional Relationship

Facts/ Characteristics

Proportional Relationships

Examples of Proportionality

4) Table

x y Ratio $\frac{y}{x}$

$\frac{y}{x}$

5) Graph

6) Equation _____

Non-Examples of Proportionality

1) Table

x y Ratio

2) Graph

3) Equation _____

Grade 8 Sample Lesson

(Proportional Reasoning)

Recommended time: 90 minutes

Domain: Expressions and Equations

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

Common Core Standard: 8.EE.6 Use similar triangles to explain why the slope *m* is the same between
any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Mathematical Practices: Make sense of problems and preserve in solving them
Construct viable arguments and critique the reasoning of others
Use appropriate tools strategically
Look for and make use of structure

Vocabulary: point-slope form

Essential Question: What does an equation in point-slope form look like and how do we graph it?

The Bridge: The cost for 1,2,3 and 4 people to go the zoo is shown in the table.

Number of People, x	1	2	3	4
Total Cost, y	$13	$22	$31	$40

Is the relationship linear? Explain.

What is the slope of the related graph?

Write an equation of the line in slope-intercept form.

Mini Lesson: The linear equation $y - y_1 = m(x - x_2)$ is written in point-slope form, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line. Notice that this really just came from the slope formula:

$\frac{y-y_1}{x-x_1} = m$ slope formula

$\frac{x-x_1}{1} \cdot \frac{y-y_1}{x-x_1} = m \cdot \frac{x-x_1}{1}$ multiplication property of equality

$\frac{\cancel{x-x_1}}{1} \cdot \frac{y-y_1}{\cancel{x-x_1}} = m \cdot \frac{x-x_1}{1}$ cross cancel

$y - y_1 = m(x - x_1)$

You can write an equation of a line in slope-intercept form when you know the slope and the y-intercept. You can write an equation of a line in **point-slope form** when you are given the slope and the coordinates of a point on the line that is not the y-intercept.

Example:
Write an equation in point-slope form for the line that passes through (-2, 3) with a slope of 4.

$y - y_1 = m(x - x_1)$ point-slope form

$y - 3 = 4(x - (-2))$ $(x_1, y_1) = (-2, 3)$, $m = 4$

$y - 3 = 4(x + 2)$ Simplify

Write the slope-intercept form of this equation.

$y - 3 = 4(x + 2)$ write the equation

$y - 3 = 4x + 8$ distributive property

$+3 \quad +3$ addition property of equality

$y = 4x + 11$ simplify

Check: Substitute the coordinates of the given point in the equation.

$y = 4x + 11$
$3 = 4(-2) + 11$
$3 = 3$

Let's put it all together: How can we write a linear equation?

from slope and a point – substitute the slope *m* and the coordinates of the point in $y - y_1 = m(x - x_1)$

from slope and y-intercept – substitute the slope *m* and y-intercept *b* in $y = mx + b$

from a graph – find the y-intercept *b* and the slope *m* from the graph, then substitute the slope and y-intercept in $y = mx + b$

from two points – use the coordinates of the points to find the slope. Substitute the slope and coordinates of one of the points in $y - y_1 = m(x - x_1)$.

from a table – use the coordinates of the two points to find the slope, then substitute the slope and coordinates of one of the points in $y - y_1 = m(x - x_1)$

The form you use to write a linear equation is based on the information you are given.

Work Period: Divide the class up into 8 groups (hopefully about 3-4 students per group). Give each group a separate worksheet. Worksheets #1- 4 have the same situations but given different pieces of information. Worksheets #5 – 8 have the same situations but given different pieces of information. When groups have completed the worksheets, have them put their work on poster paper. Have students walk around and put Post-its on other groups' posters. Each person should put 1 positive Post-It and 1 constructive criticism Post-it.

Think and Discuss

1. Think about the other 3 groups that had the same situations your group had. Even though your group was given different information (for example, maybe you were given a table but another group was given a graph, etc.) did your group come up with the same equations as the others? What piece of information (graph, table or description) would you prefer to have in order to derive the linear equation? Explain.

2. Each representation (graph, table and equation) has its advantages and disadvantages. Think of at least 1 advantage and 1 disadvantage for **each** representation.

Summary/Closure: Have students answer the essential question.

Journal Entry – In the soccer ball problem, students were told to assume that it was a linear relationship between distance the ball traveled and time. Do you think in real life this really is a linear situation? Explain.

Homework: Worksheet

Learning Extensions: Write a linear equation that is in point-slope form. Identify the slope and name a point on the line.

Name:_____ Date:_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 - (8.1.3.8) Work Period

Activity #1

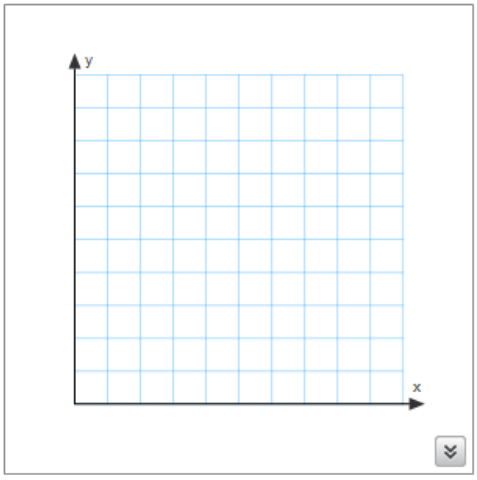
Janelle's party:

Janelle is planning a party. The cost for 20 people is $250. The cost for 70 people is $750. Write an equation in point-slope form and in slope-intercept form to represent the cost y of having a party for x people.

Point-slope form: _____

Slope-intercept form: _____

Draw a graph of the line.



Fill in the table

Number of people	Cost ($)

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How much would a party cost if she invited 60 people?

Lesson: 8.1.3.8

Science experiment:

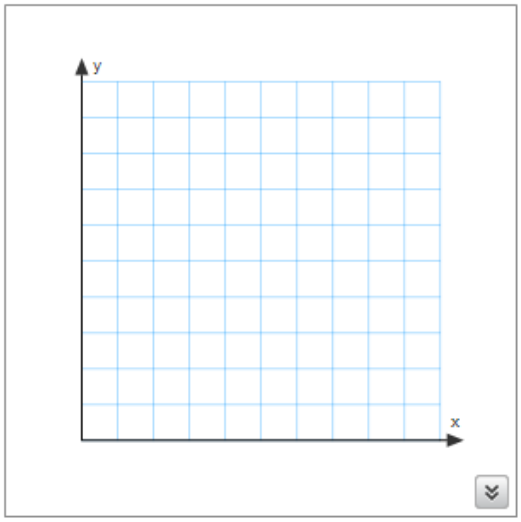
Michael measured the height of a plant every week. He recorded the information in the table. Assuming the growth is linear, write an equation in point-slope form and y-intercept form to represent the height y of the plant after x weeks.

Point-slope form: _____

Slope-intercept form: _____

Weeks	Height (In.)
5	13
10	14
15	15
20	16

Draw a graph of the line



What does the slope represent in this situation?

What does the y-intercept represent in this situation?

In how many weeks will the plant be 20 inches tall?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

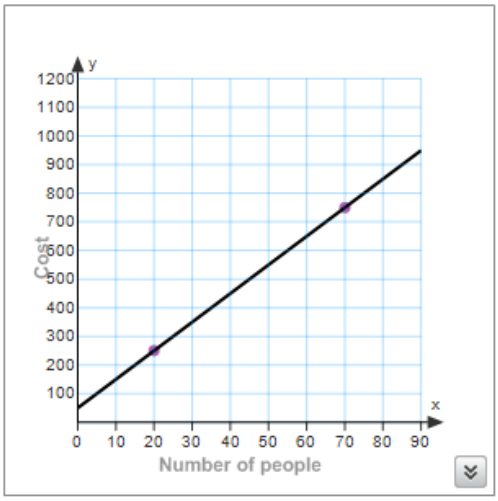
Activity #2

Janelle's party:

Janelle is planning a party. Below is a graph that represents the cost of the party vs. how many people she invites. Write an equation in point-slope form and in slope-intercept form to represent the cost y of having a party for x people.

Point-slope form: _____

Slope-intercept form: _____



Fill in a table

Number of people	**Cost ($)**

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How much would a party cost if she invited 60 people?

Name: _____ Date_____

Lesson #8 (8.1.3.8)

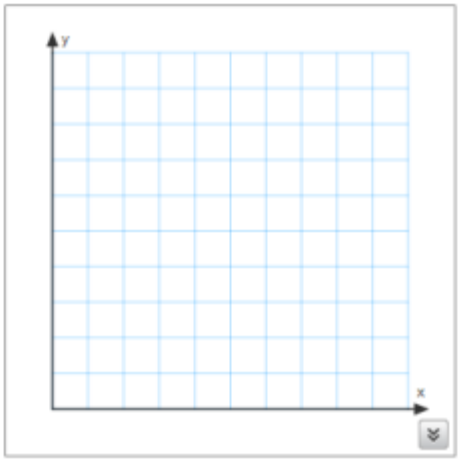
Science experiment:

Michael measured the height of a plant every week. After 5 weeks, the plant was 13 inches tall. After 10 weeks, it was 14 inches tall. Assuming the growth is linear, write an equation in point-slope form and y-intercept form to represent the height y of the plant after x weeks.

Point-slope form: _____

Slope-intercept form: _____

Draw a graph of the line



Fill in the table

Weeks	**Height (In.)**

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

In how many weeks will the plant be 20 inches tall?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

Activity #3

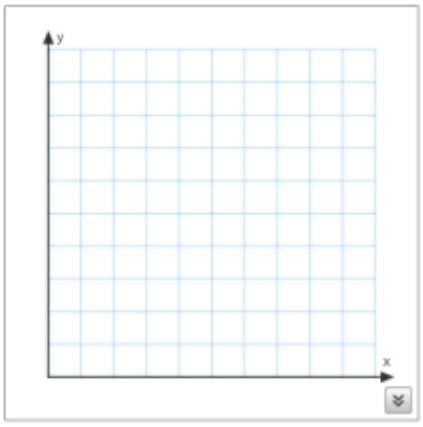
Janelle's party:

Janelle is planning a party. Below is a table that represents the cost of the party vs. how many people she invites. Write an equation in point-slope form and in slope-intercept form to represent the cost y of having a party for x people.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Number of people	Cost ($)
20	250
30	350
40	450
50	550

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How much would a party cost if she invited 60 people?

Name: _____ Date_____

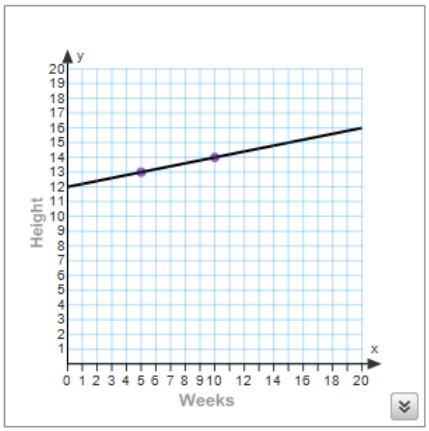
Lesson #8 (8.1.3.8)

Science experiment:

Michael measured the height of a plant every week. Below is a graph of a line of what he found. Assuming the growth is linear, write an equation in point-slope form and y-intercept form to represent the height y of the plant after x weeks.

Point-slope form: _____

Slope-intercept form: _____



Fill in the table

Weeks	**Height (In.)**

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

In how many weeks will the plant be 20 inches tall?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

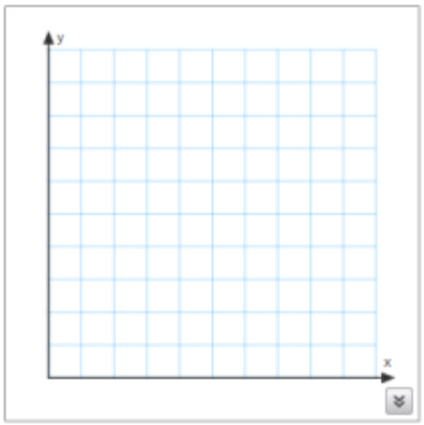
Activity #4

Janelle is planning a party. Below is a table that represents the cost of the party vs. how many people she invites. Write an equation in point-slope form and in slope-intercept form to represent the cost y of having a party for x people.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Number of people	Cost ($)
20	250
30	350
40	450
50	550

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How much would a party cost if she invited 60 people?

Name: _____ Date_____

Lesson #8 (8.1.3.8) Work Period

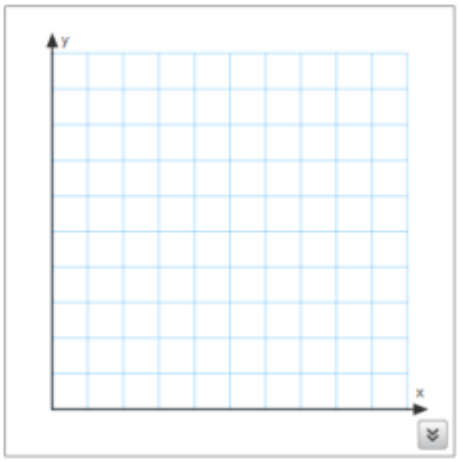
Science experiment:

Michael measured the height of a plant every week. After 5 weeks, the plant was 13 inches tall. After 10 weeks, it was 14 inches tall. Assuming the growth is linear, write an equation in point-slope form and y-intercept form to represent the height y of the plant after x weeks.

Point-slope form: _____

Slope-intercept form: _____

Draw a graph of the line



Fill in the table

Weeks	Height (In.)

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

In how many weeks will the plant be 20 inches tall?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

Activity #5

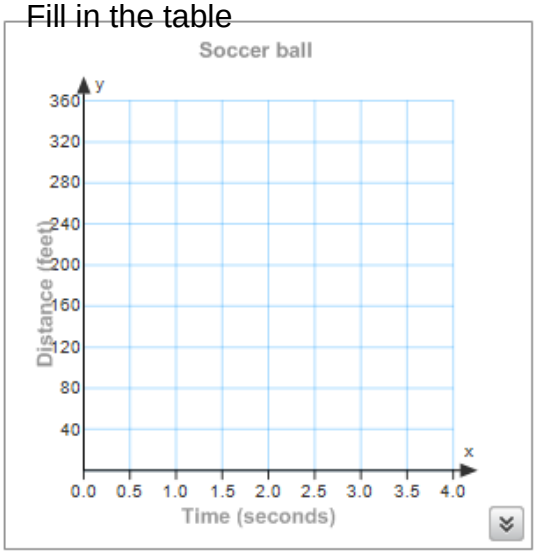
Penalty kick:

After 2 seconds on a direct kick in soccer, the ball travels 160 feet. After 2.75 seconds on the same kick, the ball travels 220 feet. Assuming this is a linear relationship, write an equation in point-slope form and slope-intercept form to represent the distance *y* of the ball after *x* seconds.

Point-slope form: _____

Slope-intercept form: _____

Graph the line
Fill in the table



Time (seconds)	Distance (feet)

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How far would the ball travel after 4 seconds?

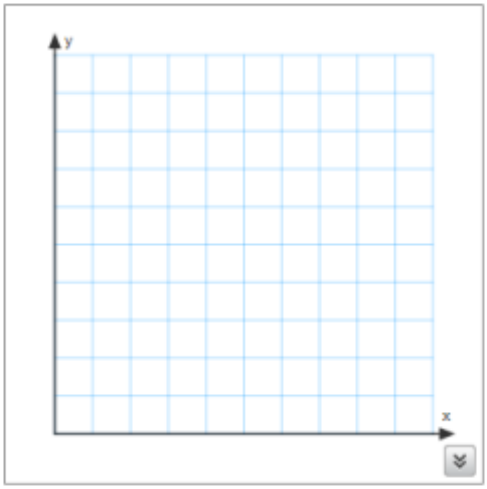
Spirit Buttons:

The cost for making spirit buttons is shown in the table. Write an equation in point-slope form and slope-intercept form to represent the cost *y* of making *x* buttons.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Number of Buttons	Cost ($)
100	25
150	35
200	45
250	55

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

Suppose you want to get spirit buttons for the entire Altmar Parish Williamstown Central School District and you figure you would need about 3,000 buttons. How much would this cost?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

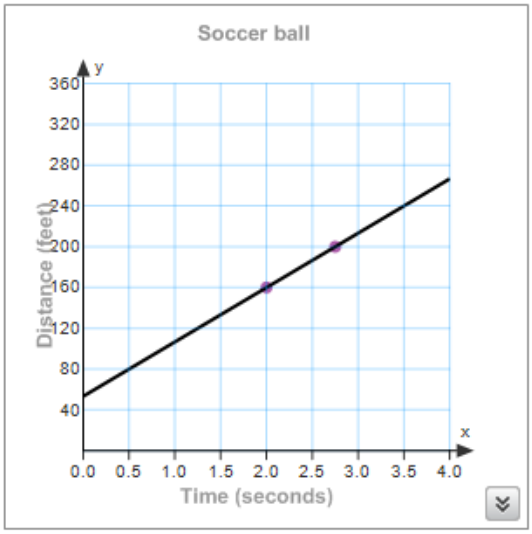
Activity #6

Soccer kick:

The graph of a line below compares a soccer ball's distance vs. time after it has been kicked. Assuming this is a linear relationship, write an equation in point-slope form and slope-intercept form to represent the distance *y* of the ball after *x* seconds.

Point-slope form: _____

Slope-intercept form: _____



Fill in the table

Time (seconds)	Distance (feet)

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How far would the ball travel after 4 seconds?

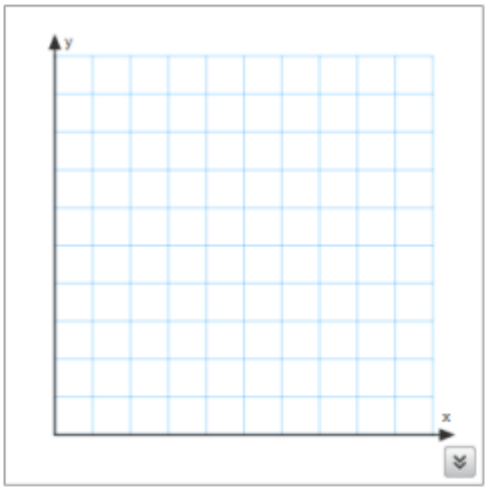
Spirit Buttons:

The cost for making spirit buttons is shown in the table. Write an equation in point-slope form and slope-intercept form to represent the cost *y* of making *x* buttons.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Number of Buttons	Cost ($)
100	25
150	35
200	45
250	55

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

Suppose you want to get spirit buttons for the entire Altmar Parish Williamstown Central School District and you figure you would need about 3,000 buttons. How much would this cost?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

Activity #7

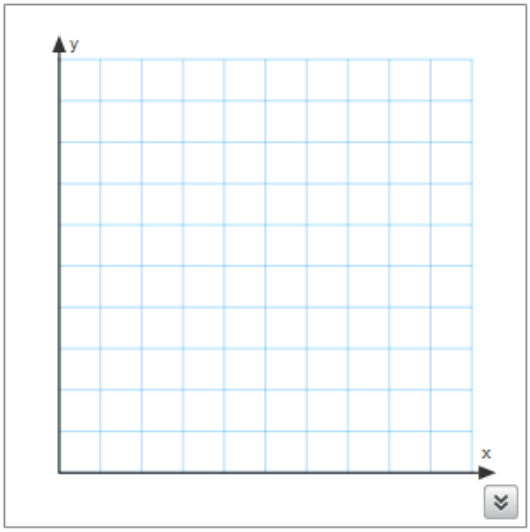
Soccer kick:

The table below represents the distance a soccer ball travels after being kicked vs. time. Assuming this is a linear relationship, write an equation in point-slope form and slope-intercept form to represent the distance *y* of the ball after *x* seconds.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Time (seconds)	Distance (feet)
1.5	120
2	160
2.5	200
3	240

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How far would the ball travel after 4 seconds?

Spirit Buttons:

Suppose you want to purchase spirit buttons. The graph below shows you the cost vs. the quantity of buttons. Write an equation in point-slope form and slope-intercept form to represent the cost *y* of making *x* buttons.

Point-slope form: _____

Slope-intercept form: _____

Fill in the table

Number of Buttons	Cost ($)

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

Suppose you want to get spirit buttons for the entire Altmar Parish Williamstown Central School District and you figure you would need about 3,000 buttons. How much would this cost?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.

Lesson #8 (8.1.3.8) Work Period

Activity #8

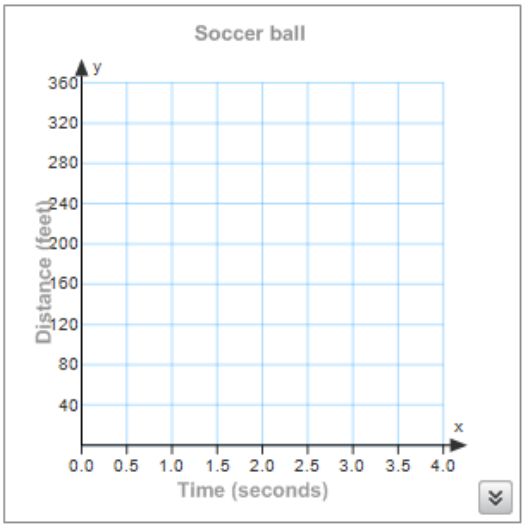
Soccer kick:

After 2 seconds on a direct kick in soccer, the ball travels 160 feet. After 2.75 seconds on the same kick, the ball travels 220 feet. Assuming this is a linear relationship, write an equation in point-slope form and slope-intercept form to represent the distance *y* of the ball after *x* seconds.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Fill in the table

Time (seconds)	**Distance (feet)**

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

How far would the ball travel after 4 seconds?

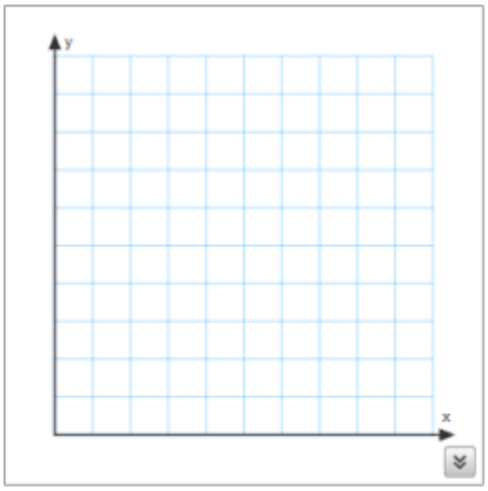
Spirit Buttons:

Suppose you want to purchase spirit buttons. A company says that will charge you $25 for 100 buttons, $35 for 150 buttons and so on. Write an equation in point-slope form and slope-intercept form to represent the cost *y* of making *x* buttons.

Point-slope form: _____

Slope-intercept form: _____

Graph the line



Fill in the table

Number of Buttons	**Cost ($)**

What does the slope represent in this situation?

What does the y-intercept represent in this situation?

Suppose you want to get spirit buttons for the entire Altmar Parish Williamstown Central School District and you figure you would need about 3,000 buttons. How much would this cost?

Name: _____ Date_____

Understand the connections between proportional relationships, lines, and linear equations.
Lesson #8 Homework

Directions: Write an equation in point-slope form and slope-intercept form for each line. Show all work.

1.) passes through (1, 9), slope = 2

2.) passes through (4, -1), slope = -3

3.) passes through (-4, -5), slope = $\frac{3}{4}$

4.) passes through (3, -6) and (-1, 2)

5.) passes through (4, - 4) and (8, -10)

6.) passes through (3, 4) and (5, -4)

Integrated Algebra Sample Lesson

Integrated Algebra Lesson Plan: System of Equations and Inequalities

Result 4

Performance Indicator: A.A.10

CCSSM Emphasis: Create equations that describe numbers or relationships (A-CED 3)

Essential Question: How do you solve verbal problems whose solution requires solving a system of equations?

The Bridge (Warm-up):

1. What is the value of the *y*-coordinate of the solution to the system of equations $x+2y=9$ and $x-y=3$?

(1) 6 (3) 3

(2) 2 (4) 5

2. What is the value of the *y*-coordinate of the solution to the system of equations $x-2y=1$ and $x+4y=7$?

(1) 1 (3) 3

(2) -1 (4) 4

The Mini Lesson:

- Model the following problem:

1. Jack bought 3 slices of cheese pizza and 4 slices of mushroom pizza for a total cost of $12.50. Grace bought 3 slices of cheese pizza and 2 slices of mushroom pizza for a total cost of $8.50. What is the cost of one slice of mushroom pizza?
2. The sophomore class at South High School raised $800 from the sale of tickets to a dance. Tickets sold for $1.50 in advance and $2.00 at the door. If a total of 475 tickets were sold, what was the number of tickets sold at the door?

Work Period

- See attached worksheet

Summary/ Closure:

1. The cost of 3 markers and 2 pencils is $1.80. The cost of 4 markers and 6 pencils is $2.90. What is the cost of each item? Include appropriate units in your answer.
2. Ramón rented a sprayer and a generator. On his first job, he used each piece of equipment for 6 hours at a total cost of $90. On his second job, he used the sprayer for 4 hours and the generator for 8 hours at a total cost of $100. What was the hourly cost of *each* piece of equipment?

Homework:

- See attached work period PDF from Above
- Apprentice Problems Attached below as a word document
- More examples can be found at jmap.org or http://www.regentsprep.org/Regents/math/ALGEBRA/AE3/PracWord.htm

Integrated Algebra Name: _____

Algebraic Systems

1.	The freshman class at Boomtown High School raised $930 from the sale of tickets to the spring dance. Tickets were $3.50 if purchased in advance and $5.00 if purchased at the door. If 225 tickets were sold, what is the total number of tickets sold at the door? [*Show or explain the procedure used to obtain your answer.*] [10]
2.	Cedric and Zelda went shopping at Price Buster. Cedric bought 2 jumbo rolls of aluminum foil and 3 packages of AA batteries for a total cost of $21. Zelda bought 5 identical jumbo rolls of aluminum foil and 2 identical packages of AA batteries for a total cost of $25. Find the cost of 1 roll of aluminum foil and find the cost of 1 package of AA batteries. [*Only an algebraic solution will be accepted.*] [10]

3.	The Town Recreation Department ordered a total of 100 balls and bats for the summer baseball camp. Balls cost $4.50 each and bats cost $20.00 each. The total purchase was $822.00. How many of each item were ordered? [*Use any method to arrive at your answer. Show all work.*] [10]
4.	The senior class at Northwest High School needed to raise money for the yearbook. A local sporting goods store donated hats and T-shirts. The number of T-shirts was three times the number of hats. The seniors charged $5 for each hat and $8 for each T-shirt. If the seniors sold everything and raised $435, what was the total number of hats and the total number of T-shirts that were sold? [*Show or explain the procedure used to obtain your answer.*] [10]

Geometry Sample Lesson

Geometry
Concepts of Density

CCSSM: G.MG. Apply concepts of density based on area and volume in modeling situations (e.g.persons per square mile, btu's per cubic foot)

45 -90 minutes estimated

Warm Up: (10 minutes)

There will be a concert at school and the audience will be allowed to stand on the football field that measures 120 yards long(including end zones) and 70 yards wide. There will be no other seating. You want to sell enough tickets to make a sizeable profit, but you want to ensure the safety of the audience and not overcrowd. What is a reasonable number of tickets to sell? Explain your answer clearly.

The Mini Lesson: Conceptual Understanding

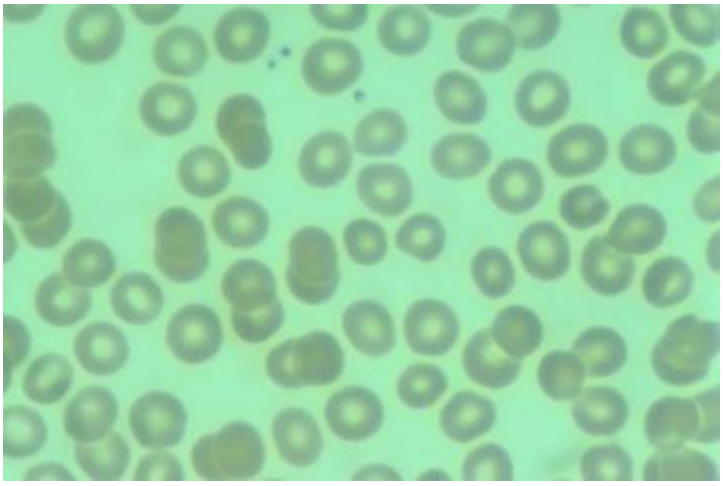
Allow students to explain their thinking from the Warm Up activity. Try to move students to explain their solution in terms of a safe number of people per square yard or even per square foot. If you use people per square yard, encourage students to discuss how they decide what is a safe number of people to fit - Did they use width of shoulder of a typical person?

Allow multiple students to demonstrate their solution and encourage other students to ask questions to presenters.

Work Period: Application
With your partner, complete the following problem:

About how many cells are in the human body ?

You can assume that a cell is a sphere with radius 10^{-3} cm and that the density of a cell is approximately the density of water which is $1 g/cm^3$.



Teacher Notes:
The purpose of this task is for students to apply the concepts of mass, volume, and density in a real-world context. There are several ways one might approach the problem, e.g., by estimating the volume of a person and dividing by the volume of a cell. The main pitfall with that approach is that students generally know how much a person weighs, but are less likely to make accurate estimates of a person's volume. The task provides an opportunity to think about attention to mathematical precision. Note that despite maintaining several digits of accuracy throughout the calculation, we report an answer with only one significant digit.

Students may need to be reminded that mass = density x volume. And one pound is approximately 454 grams.

After students have worked on this problem, teachers should spend some time discussing the reasonableness of the assumptions provided. Cells are not really spherical, for example, but getting the right order of magnitude for the volume is probably sufficient for this type of estimation. (For example, if we replace our spherical model of a cell by a cubical model, our net estimate will be cut approximately in half). Also, different cells are likely to have different densities and are not all packed together as tightly in all parts of the body (consider bone cells, for example). This task could be nicely paired with work in a biology class where students could discuss these issues in more depth.

Summary/ Closure:
Allow students to present multiple solutions. Be sure to discuss the enormity and magnitude of the number of cells. It turns out that a 100 lb person has approximately ten trillion cells. Ten trillion is a very interesting number to discuss!

Extension:
What other real life applications depend on density of area or volume. Write out several examples.

Homework:

You have been hired by the owner of a local ice cream parlor to assist in his company's new venture. The company will soon sell its ice cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use a real ice cream cone or the dimensions of a real ice cream cone to complete the following tasks.

- Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly.
- Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of length, r, and a slant height, s.
- Using measurements of the radius of the base and slant height of your cone, and your equation from the previous step, find the surface area of your cone.
- The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.

Algebra 2 with Trigonometry Sample Lesson

Algebra 2 with Trig
Modeling Periodic Behavior

CCSSM: F.IF. Model periodic phenomena with trigonometric functions

Essential Question: How do you use trigonometric functions to model periodic behavior?

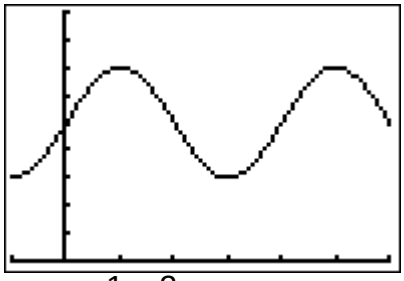
(Warm-up): Fluency

For each of the following functions, find the amplitude, period, and phase shifts.

1. $y = 5\ sin\ 2x + 3$
2. $y = 2\ cos\ \frac{1}{2}(x - 2) - 1$
3. $y = cos\ \frac{\pi}{2}(x - 1) + 3$
4. $y + 3 = 6\ sin\ \frac{\pi}{4}(x + 2)$

The Mini Lesson: Fluency and Conceptual Understanding

To answer questions 1-5, refer to the picture of the trigonometric function below. Scale increase by 1



1. What is the amplitude?

2. What is the period?

3. What is the axis of the wave?

4. If the wave is considered to be a translation of the sine graph, what are the horizontal and vertical translation amounts? What is the sine equation?

5. If the wave is considered to be a translation of the cosine graph, what are the horizontal and vertical translation amounts? What is the cosine equation?

Work Period: Application
With your partner, complete the following problem. An accurate graph must be .

You are sitting on a pier watching the waves when you notice a bottle in the water. The bottle bobs so that it is between 2.5 ft and 4.5 ft below the pier. You know you can reach 3 ft below the pier. Suppose the bottle reaches its highest point every 5 s.

a. Sketch a graph of the bottle's distance below the pier for 15 s. Assume that at $t = 0$, the bottle is closest to the pier.

b. Find the period and the amplitude of the function. Write the equation of the function.

c. Estimation Use your graph to estimate the length of time the bottle is within reach during each cycle. Explain your answer in context.

Extension:
The function $y = 1.5 \sin\frac{\pi}{6}(x - 6) + 2$ represents the average monthly rainfall for a town in Central Florida, where x represents the number of the month. (January = 1, February =2, etc…) Rewrite the function using a cosine model.

Summary/ Closure:
Sketch the graph: $y = -3\sin(x - \frac{\pi}{6})$. Identify the amplitude, period, and phase shifts.

Homework:
- Page 872, 7, 13, 15, 17, 24, 27, 46 Graph paper is needed.

Calculus Sample Lesson

Calculus
Applications of Optimization

45 minutes estimated

Warm Up: (10 minutes)

The function $f(x) = x^2 - ax + b$ has a relative minimum at x = 2. Find a.

Minilesson:

GUIDELINES FOR SOLVING MAX./MIN. PROBLEMS

1. Read each problem slowly and carefully. Sometimes words can be ambiguous. It is imperative to know exactly what the problem is asking. If you misread the problem, you will have difficulty solving it correctly.
2. If appropriate, draw a sketch or diagram of the problem to be solved. Pictures are a great help in organizing and sorting out your thoughts.
3. Define variables to be used and carefully label your picture or diagram with these variables. This step is very important because it leads directly or indirectly to the creation of mathematical equations.
4. Write down all equations which are related to your problem or diagram. Clearly denote that equation which you are asked to maximize or minimize.
5. Verify that your result is a maximum or minimum value using the first or second derivative test for extrema.

Example:
How can 20 be partitioned such that the product of one part and the square of the other part shall be a maximum

Worktime:

It costs a manufacturer C dollars per item to make and distribute the item. If they sell each item for x dollars, it is estimated that the number N sold is given by the equation:

N = $\frac{A}{x-C}$ + B(100 – x) where A and B are positive constants.

What selling price will bring the maximum profit, if profit is represented by p = xN – CN

Teacher Notes: Please notice that C is also a constant. This task will require solid algebraic manipulation skills. The solution will not be numeric.

Extension:

A window has the outline of a semicircle on top of a rectangle. Suppose that there is 8 + π feet of wood trim available. Discuss why a window designer might want to maximize the area of the window. Find the dimensions of the rectangle that will maximize the area of the entire window.

In order to get ready for the New Common Core Mathematics tests below are suggested activities and resources that can be utilized during the extra period Mathematics block. Students who do not have a double period scheduled may utilize this in intervention classrooms or after school tutorial in order to prepare for the tests.

Components	Instructional Methods
Fluency (10 minutes)	• Interventions at all grade levels should devote time in each session to build fluent retrieval of basic arithmetic facts. • This can be done with flash cards, timed tests, fames, or any other routine
Tasks and Practice (35 minutes)	• Instruction during the intervention block should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review. • Intervention should include instruction on solving word problems that is based on common underlying structures. • Students should have opportunities to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Grade 6

Weeks	Standards	Fluencies	Tasks and Resources
2/11-3/1	• 6.NS.2 • 6.NS.6 • 6.NS.8 • 6.G.3	• Multiplication and Division	• APW Common Core Units • Got Your Number • Basketball Players • Once Upon a Time
	• 6.NS.3 • 6.G.1	• Operations with Decimals	• APW Common Core Units • Sewing
3/4-3/22	• 6.NS.1 • 6.RP	• Operations with fractions	Understand Ratio concepts and use ratio reasoning to solve problems • http://illustrativemathematics.org/illustrations/496 • http://illustrativemathematics.org/illustrations/61 • http://illustrativemathematics.org/illustrations/62

			• http://illustrativemathematics.org/illustrations/63 • http://illustrativemathematics.org/illustrations/64 • Grade 6 Performance Task: Taking a Field Trip • Grade 6 Math: Ratios and Proportional Relationships (NYC) • Grade 6 Math: Ratio Reasoning (NYC) • Candies • Truffles • First Rate • Movin n Grovin • Apply and extend previous understandings of multiplication and division to divide fractions by fractions • http://illustrativemathematics.org/illustrations/463 • http://illustrativemathematics.org/illustrations/330 • http://illustrativemathematics.org/illustrations/409 • Rabbit Costumes

Educator Guide to the 2013 Grade 6 Common Core Mathematics Test
http://illustrativemathematics.org/
Mathematics Assessment Project
Mathematics Grade 6 Common Core Sample Questions

Additional Tasks from Rochester's Curriculum
www.learnzillion.com
Middle School Balanced Assessment Tasks
Inside Mathematics – Grade 6

Grade 7

Weeks	Standards	Fluencies	Tasks and Resources
2/8-3/1	• 7.NS.1-2 • 7.RP	• Addition and Subtraction with rational numbers	• http://illustrativemathematics.org/illustrations/98 • http://illustrativemathematics.org/illustrations/99 • http://illustrativemathematics.org/illustrations/82 • Buses • Sale! • T-Shirt Sale • A Golden Crown? • Counting Trees • Ice Cream Lessons: • Increasing and decreasing a number by a percent (lesson and ppt) • Estimating: Counting Trees (lesson and ppt)
3/4-3/22	• 7.NS.1-2 • 7.NS	• Multiplication and division with rational numbers	• http://illustrativemathematics.org/illustrations/314 • http://illustrativemathematics.org/illustrations/310 • http://illustrativemathematics.org/illustrations/46 • http://illustrativemathematics.org/illustrations/604 • http://illustrativemathematics.org/illustrations/298 • Division • A Day Out • Taxi Cabs • Using positive and negative numbers in context (lesson and resources)
3/25-	• 7. EE.4 • 7.EE	• px + q = r	Use properties of equations to generate equivalent expressions

4/19	• p(x + q) = r	• http://illustrativemathematics.org/illustrations/543 • http://illustrativemathematics.org/illustrations/433 • http://illustrativemathematics.org/illustrations/541 Solve real-life and mathematical problems using numerical and algebraic expressions and equations. • http://illustrativemathematics.org/illustrations/712 • http://illustrativemathematics.org/illustrations/478 • http://illustrativemathematics.org/illustrations/108 • http://illustrativemathematics.org/illustrations/643 • http://illustrativemathematics.org/illustrations/986 Steps to solving equations (Lesson and ppt) Fencing

Educators Guide to the 2013 Grade 7 Common Core Mathematics Test
Mathematics Assessment Project
http://illustrativemathematics.org/
Middle School Balanced Assessment Tasks
Inside Mathematics – Grade 7
www.learnzillion.com
Mathematics Grade 7 Common Core Sample Questions
Grade 7 Math: Proportional Reasoning (NYC)

Grade 8

Weeks	Standards	Fluencies	Tasks and Resources
2/8-3/1	• 8.EE.8	• Solve simple 2x2 systems by inspection	• http://www.illustrativemathematics.org/illustrations/469 • http://www.illustrativemathematics.org/illustrations/472 • http://www.illustrativemathematics.org/illustrations/554 • http://www.illustrativemathematics.org/illustrations/73 • Buying Chips and Candy • Hot Under the Collar Lessons: • Building and Solving Equations 1 (lesson and ppt) • Classifying Solutions to Systems of Equations (lesson and ppt)
3/4-3/22	• 8.F.1 • 8.F.2 • 8.F.3 • 8.F.4	• Define and Evaluate Functions • Real-life Models	Define, Evaluate, and Compare • http://www.illustrativemathematics.org/illustrations/713 • http://www.illustrativemathematics.org/standards/k8 • http://www.illustrativemathematics.org/illustrations/1165 • http://www.illustrativemathematics.org/illustrations/641 • http://www.illustrativemathematics.org/illustrations/813 Use Functions to model Relationships: Real-life • http://www.illustrativemathematics.org/illustrations/417 • http://www.illustrativemathematics.org/illustrations/552 • http://www.illustrativemathematics.org/illustrations/477 • http://www.illustrativemathematics.org/illustrations/1206 • http://www.illustrativemathematics.org/illustrations/247

			• Baseball Jerseys • Short tasks using Functions Lessons: • Interpreting Distance-Time Graphs (lesson and ppt) • Modeling Situations with Linear Equations (lesson and ppt)
3/22-4/19	• 8.EE.4 • 8.EE.3	• **Scientific Notation** • Estimating with Scientific Notation • Perform operations with scientific notation	Solve real-life and mathematical problems using scientific notation • http://www.illustrativemathematics.org/illustrations/476 • http://www.illustrativemathematics.org/standards/k8 • http://www.illustrativemathematics.org/illustrations/113 • 100 People • A Million Dollars Lessons: • Estimating Length Using Scientific Notation

Mathematics Grade 8 Common Core Sample Questions
Educators Guide to the 2013 Grade 8 Common Core Mathematics Test
Mathematics Assessment Project
http://illustrativemathematics.org/
Middle School Balanced Assessment Tasks
www.learnzillion.com
http://insidemathematics.org